Exploring Generic Heterogeneous Implementations of Graph Algorithms

Projecto Integrado CPD

Cristiano Sousa
Department of Informatics
University of Minho

Artur Mariano
Institute for Scientific Computing
Technische Universität Darmstadt

Universidade do Minho

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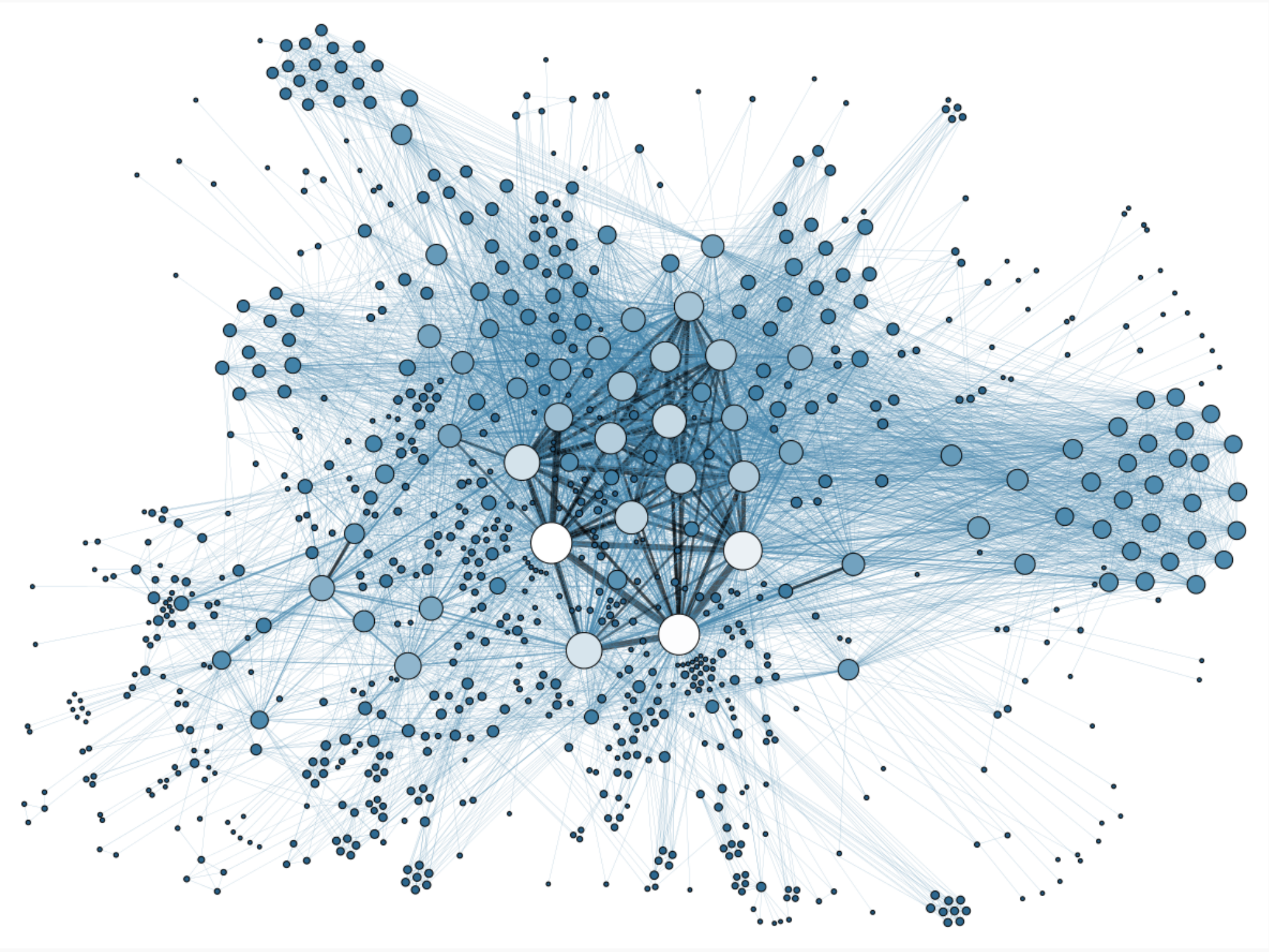
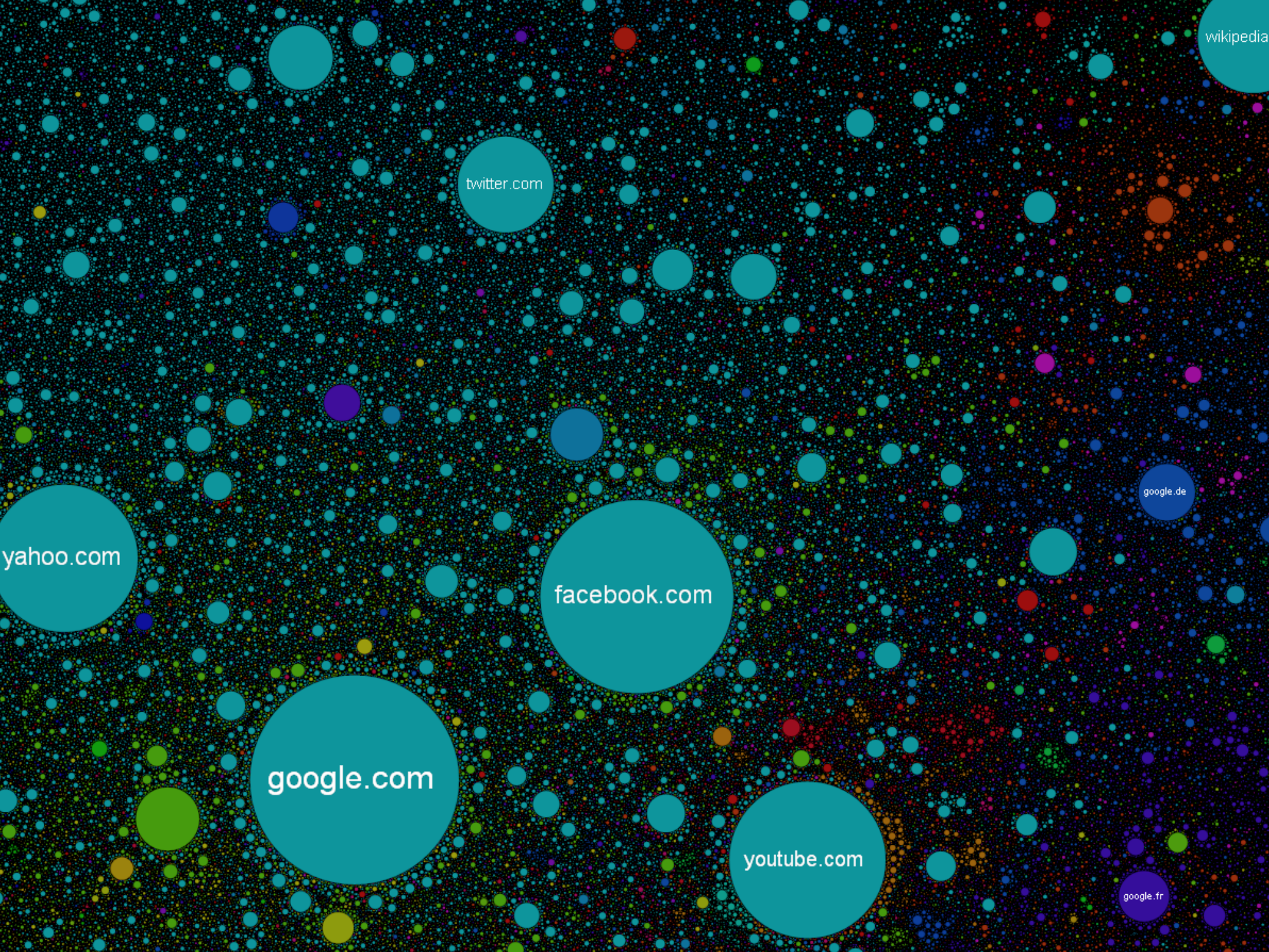
What are Graphs?

- Set of vertices that can be connected through edges
- Vertices can represent any object or entity
- Edges define any type of relationship between vertices

Graphs are important data structures, used in real-world problems

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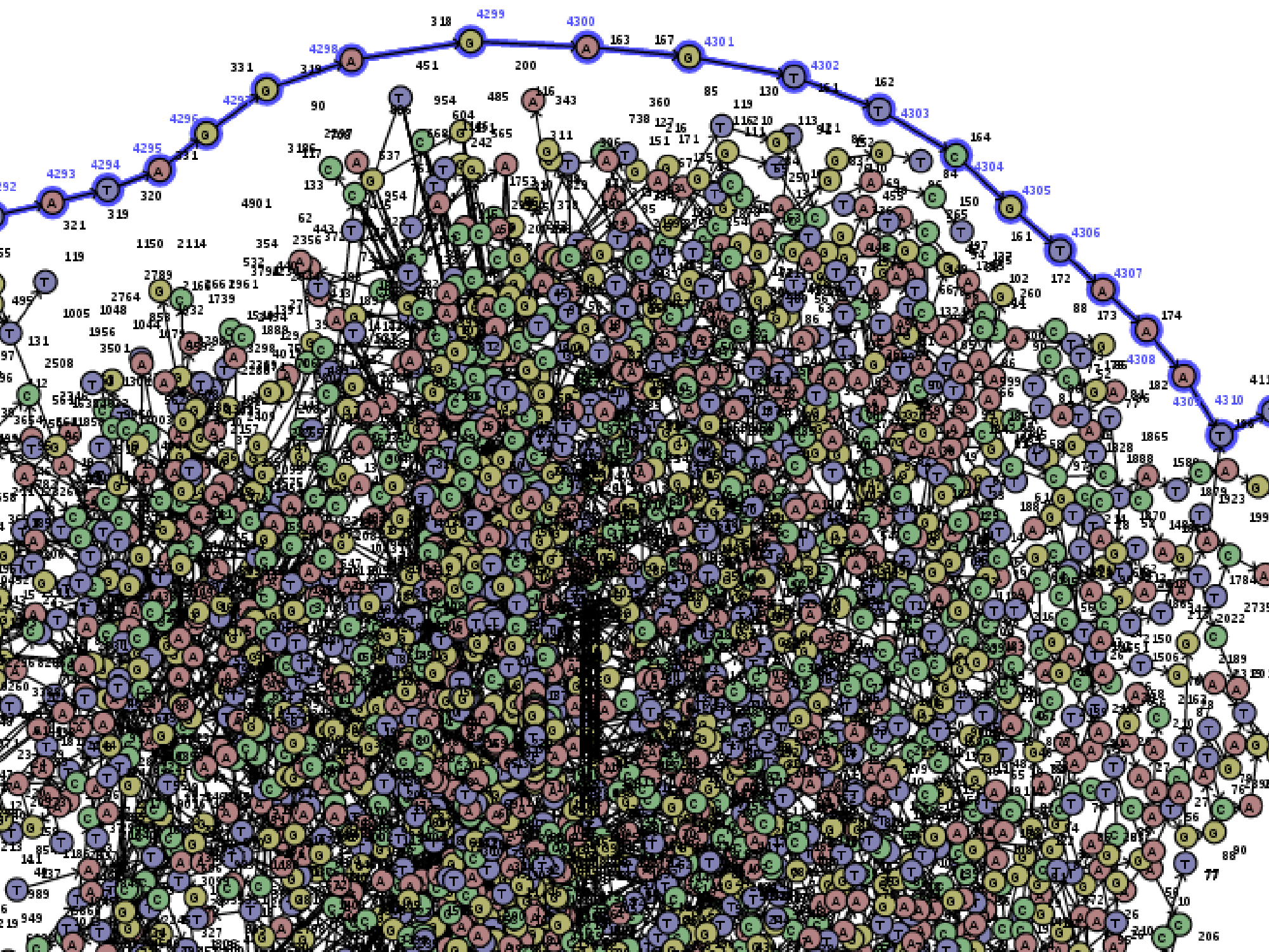
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Graph Algorithms In Current Problems

- Graph Partitioning
  - To deal with work-load distribution of large graphs
- Minimum Spanning Tree
  - DNA sequencing, Electroencephalographies and other medical studies
  - Used in the approximation of other graph algorithms
- Betweenness Centrality
  - Metrics on any network represented as a graph, to provide QoS, etc.
  - Social network analysis

The challenge:

- (Memory)-Irregular class of algorithms
  - Why?

Adjacency Matrix

- Value at position (i, j) represents an edge from i to j with the specified weight

	a	b	c	d	e	f	g
a		4		12	9		
b	4		8		7		
c		8			5		2
d	12				3	6	
e	9	7	5	3		1	13
f				6	1		11
g			2		13	11	

Figure: Adjacency matrix representation.

- Pros: $\mathcal{O}(1)$ read and insertion and removal of edges
- Cons: $\mathcal{O}(|V| \times |V|)$ memory requirements, $\mathcal{O}(|V|)$ edge iteration per vertex, stores non-existing edges

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Memory usage for large graphs

For example, a graph with 100.000 vertices would use $100.000^2 \times 4$ bytes $\approx$ 37GB of RAM

Adjacency List

- Array of vertices
- Each entry points to a list of destination vertex and weight pairs

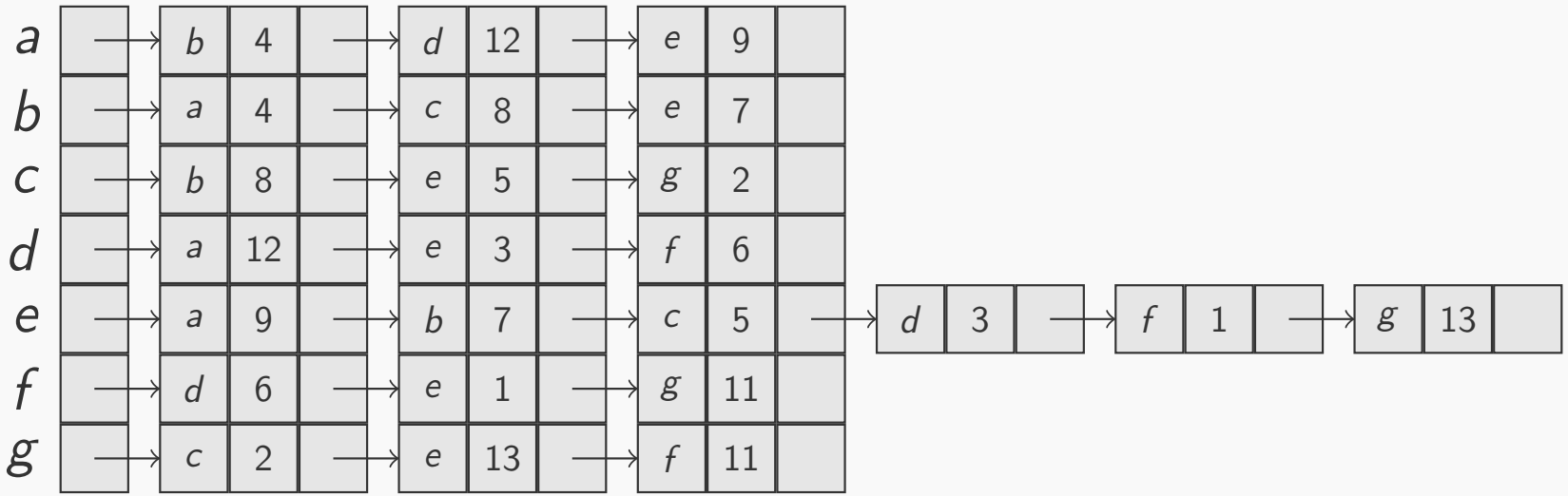

Figure: Adjacency list representation.

- Pros: $\mathcal{O}(1)$ insertion of edges, mutable, efficient edge iteration
- Cons: Heavy usage of linked-lists and pointers, costly edge lookup

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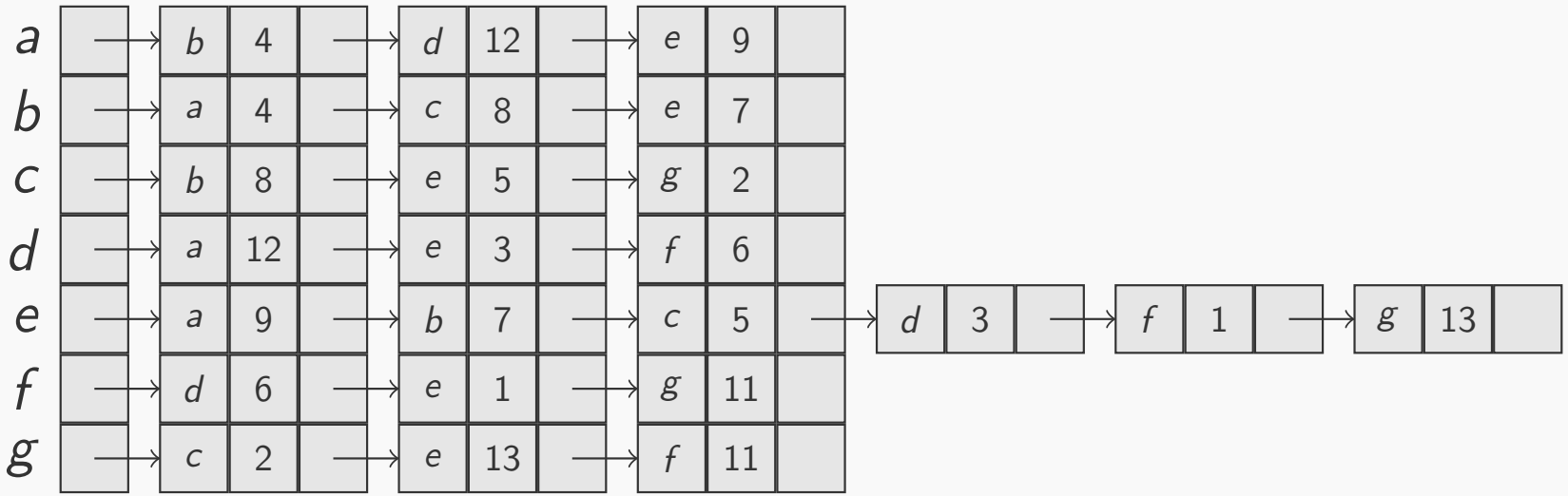

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Linked-lists?

Instead of linked-lists, arrays can be used, sacrificing mutability for a more cache friendly approach.

Generic Approach

How can we obtain a generic solution?

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- Graph representation
  - Adjacency Lists are not suited for the GPU
  - Large memory requirements of the Adjacency Matrix limit graph sizes

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$\hookrightarrow$ Unified graph representation

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- Algorithm
  - Different platforms often imply distinct implementations and parallelization approaches
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$\hookrightarrow$ Generic algorithm

and respective platform-independent implementation

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- Scheduling and Parallelization
  - Usual route of implementing an algorithm and then parallelizing computationally intensive routines is a no go
  - Algorithms must be conceptualized for heterogeneous implementations, from the ground up

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$\hookrightarrow$ Cross-platform parallelization and scheduling approach

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↪ Unified graph representation

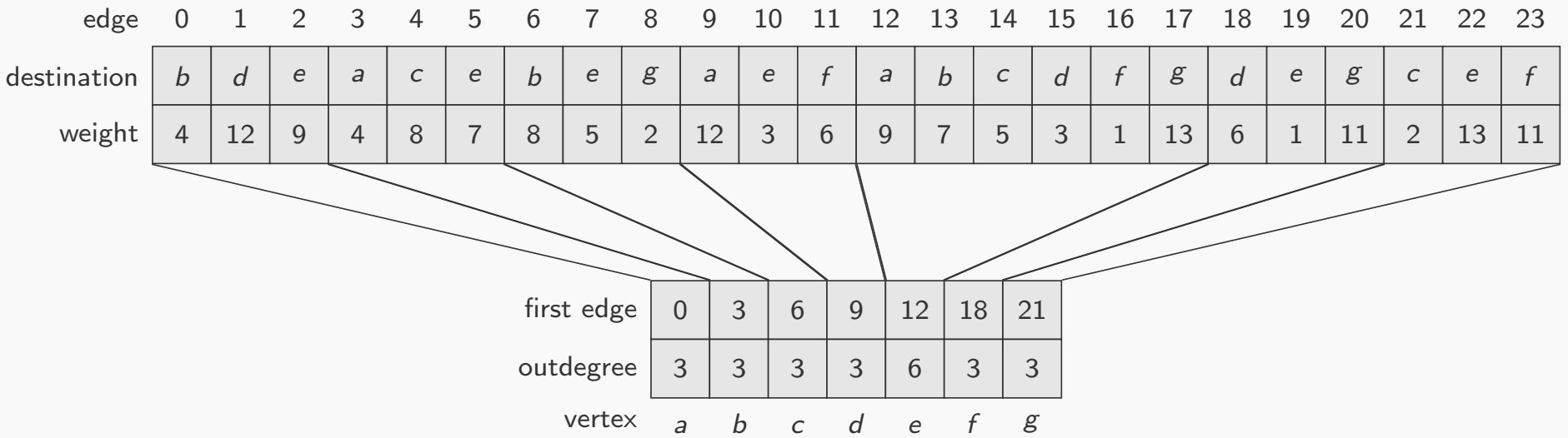
↪ Generic algorithm
and respective platform-independent implementation

↪ Cross-platform parallelization and scheduling approach

Unified Graph Representation

Compressed Sparse Row (CSR) format

- Structure of Arrays (SoA)
- Used to represent sparse matrices
- Often seen in the literature as the representation of choice for graph algorithms on the GPU



Generic Algorithm and Scheduling

Traditional approach:

```
1: while there is work to be done do
2: |   for each work item do
3: |   |   Do stuff...
4: |   Some other stuff
5: |   ...
6: |   for each work item do
7: |   |   Do more stuff...
```

Generic Algorithm and Scheduling

- Topological scheduling: each vertex is a work unit
- Operator based parallelization: operators are applied to each vertex
  - The operator has to be a well defined set of instructions that can be applied to all active vertices, at the same time

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1: **while** there are active vertices **do**
2: | Apply **operator** 1
3: | Synchronize
4: | Apply **operator** 2
5: | Synchronize
6: | ...
7: | Apply **operator** N
8: | Synchronize
9: | Determine active vertices

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Generic Implementation

The body of the operator should be the same across multiple computing platforms. Only synchronization should differ.

The Minimum Spanning Tree Problem

- Minimum Spanning Tree?
  - Well known graph problem
  - Used in Medicine, Biomedics, social-, road- and compute-networks
- To solve the MST-problem, several algorithms have been proposed
  - Borůvka, Kruskal and Prim
- Sequential implementations focus on data structures
- Current parallel implementations are too demanding and have limitations
- (Memory)-Irregular class of algorithms

Example Minimum Spanning Tree

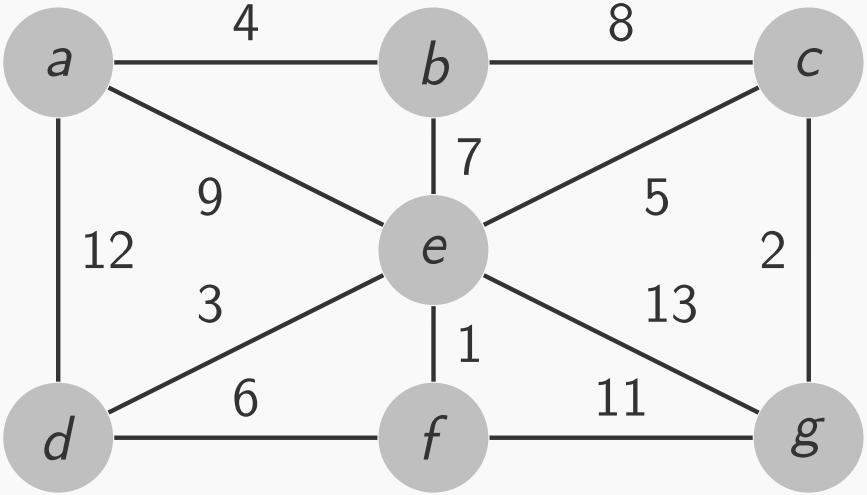

Figure: Example graph.

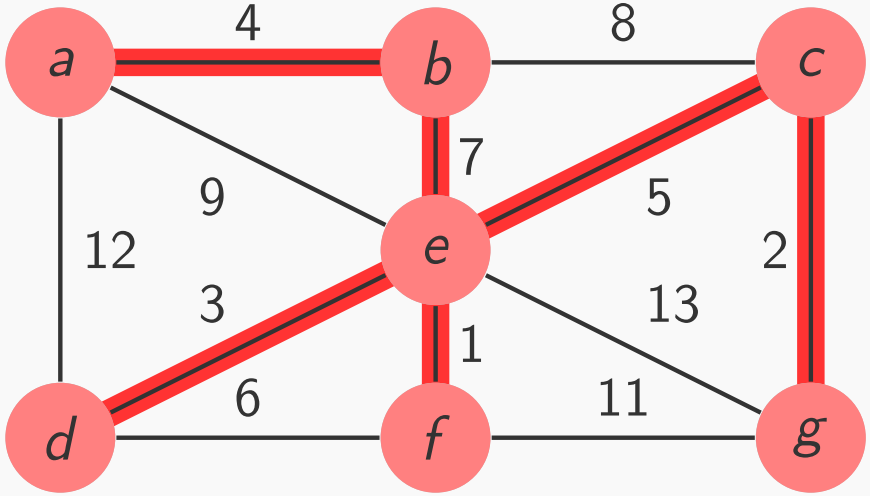

Figure: MST of the example graph.

Borůvka's Algorithm

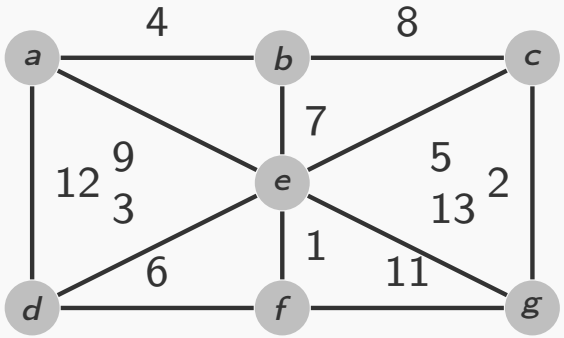

Figure: Initial graph.

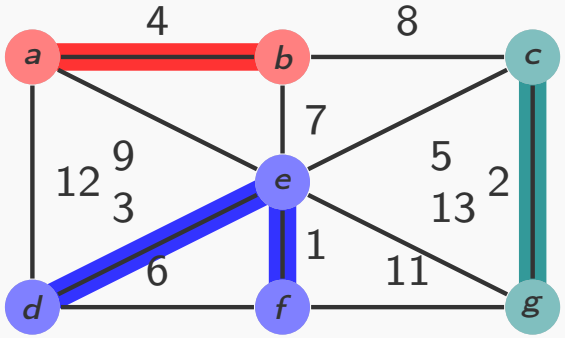

Figure: Find lightest edges.

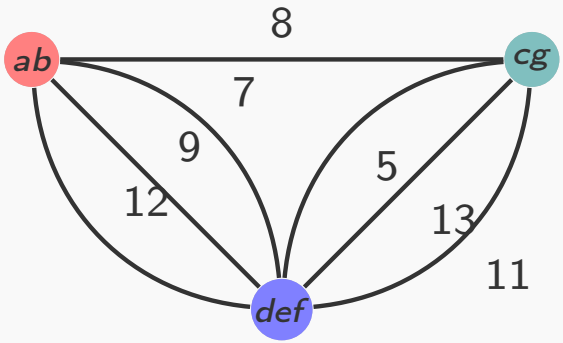

Figure: Contract.

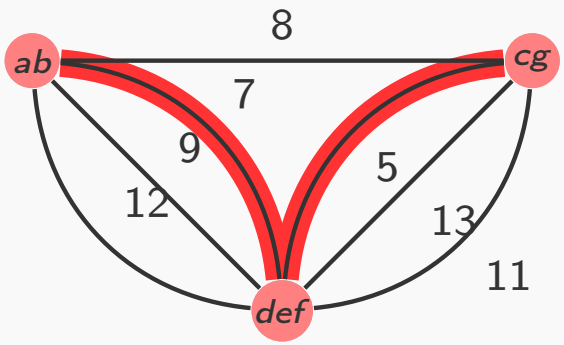

Figure: Find lightest edges.

Figure: Contract.

Algorithm 1 Parallel Generic Borůvka

Input: Undirected, connected and weighted graph $G(V, E)$

```
1:  while number of vertices > 1 do
2:  |   Find minimum edge per vertex
3:  |   Remove mirrored edges
4:  |   Initialize colors
5:  |   while not converged do
6:  |   |   Propagate colors
7:  |   Create new vertex ids
8:  |   Count new edges
9:  |   Assign edge segments to new vertices
10: |   Insert new edges
```

Project Proposal

What will you be working on?

- Assess the performance of the Generic Borůvka implementation
- Explore the generic approach to other graph algorithms
- Library for high performance, generic, heterogeneous, graph algorithms

Resources:

- MSc. Thesis
- Publication
- Generic Borůvka source code
- Artur and me

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