Finite volume method for tsunami simulation

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Collaboration with the IPMA

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The different tsunamis

☞ Tsunami :Japanese ; from *tsu*, a harbour + *nami*, a wave
☞ It is a large sea wave caused

by an

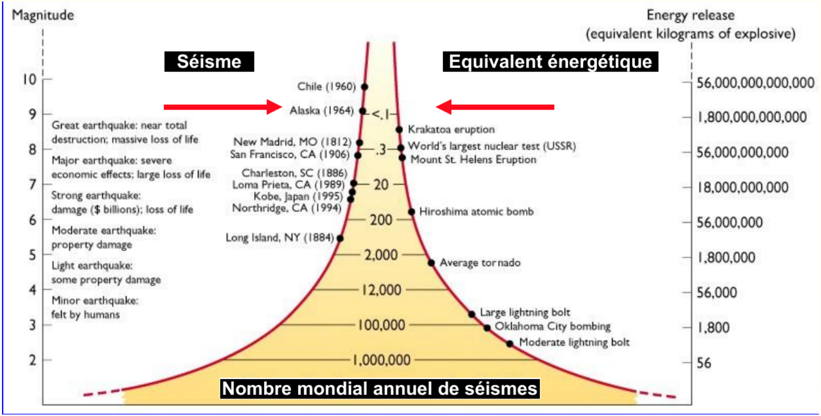
- earthquake (Lisbon 1755)
- landslide (Madeira 1930)
- volcano (Krakatoa in Indonesia 1883)
- other disturbance under the ocean

Deadly phenomenon: Japan (2011) 22.000 dead persons, Indonesia (2004), 230.000 dead persons

Numerical simulations

Prediction and scenario analysis to design infrastructures and save life. Determine the safe zones in coastal regions.

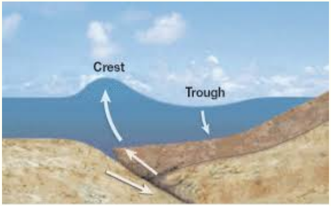
Tsunami energy

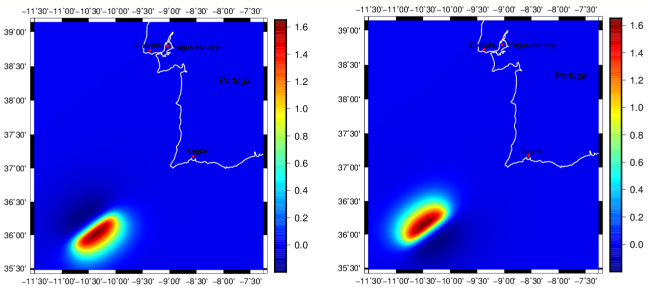


- Hiroshima: 18 kilotons • Indonesia 2004: 26 megatons
- Japan 2011: 45 megatons • Chile 1960: 160 megatons

The generation

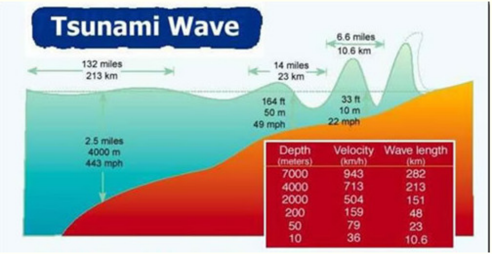
- Reverse faults: two plates collide and one plate is lifted over the other plate
- on one side: a column of water is lifted (crest)
- on the other side: a column of water grows hollow (trough)

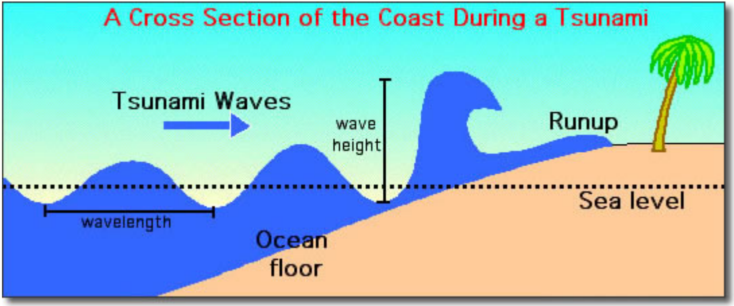




The Shoaling Process

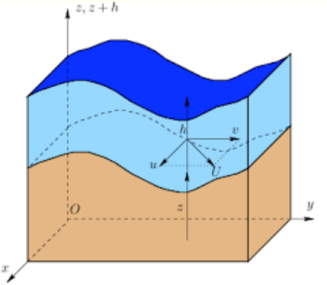
- Velocity $c = \sqrt{gh}$
- high velocity, small height, long length in deep ocean
- low velocity, large height, small length in shallow water





The modelling

- h water height
- b bathymetry
- $\eta = b + h$ free surface
- $U = (u, v)$ horizontal velocities



reduced model

Integration over a water column and neglecting the vertical velocity, the Navier-Stokes system provides the shallow water equations

The shallow water system

$$\begin{aligned} \partial_t h + \partial_x(hu) + \partial_y(hv) &= 0, \\ \partial_t(hu) + \partial_x(hu^2 + gh^2/2) + \partial_y(huv) &= -gh\partial_x b - k\frac{h|U|u}{h^\eta}, \\ \partial_t(hv) + \partial_x(hvu) + \partial_y(hv^2 + gh^2/2) + &= -gh\partial_y b - k\frac{h|U|v}{h^\eta}. \end{aligned}$$

- hu, hv mass flow, $|U| = \sqrt{u^2 + v^2}$ velocity norm
- $gh^2/2$ hydrostatic pressure
- $gh\partial_x b$, $gh\partial_y b$ gravity force
- $k\frac{h|U|u}{h^\eta}$, $k\frac{h|U|v}{h^\eta}$ friction force (Manning law).

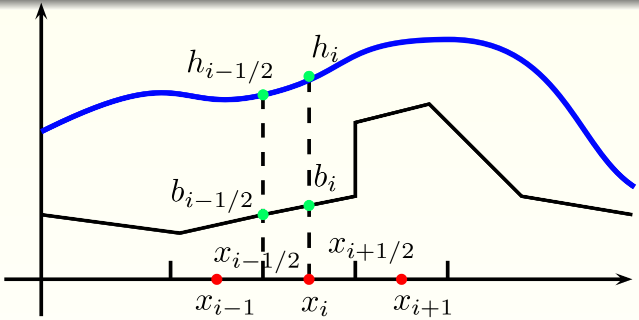
Numerical methods

• Finite difference: used in the 70's until now due to its simplicity. A lot of drawbacks, not mass conservative, wrong shock propagation, high viscosity for stabilization, second-order with non-physical oscillations

• Finite element: used in the 90's. not mass conservative per cell, viscosity for stabilisation, complex finite element basis (hyperbolic problem)

• Finite volume: mass preservation, second-order easy to achieved, correct shock propagation, no oscillations. Very good for the S-W system,

1D discretization



cells $c_i = [x_{i-1/2}, x_{i+1/2}]$ with centroid x_i and interface $x_{i+1/2}$

h_i^n, u_i^n, η_i^n, b_i approximations on cell c_i at time t^n

$W_i^n = (h_i^n, h_i^n u_i^n)$ conservative variables vector

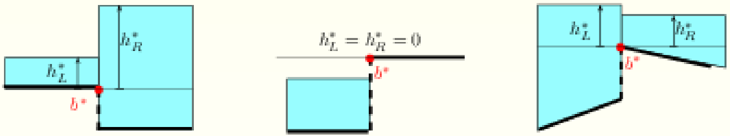
Finite volume scheme

generic scheme

$$W_i^{n+1} = W_i^n - \frac{\Delta t}{\Delta x_i}\left[\mathcal{F}_{i+1/2}^n + \varepsilon_{i+1/2,L}^n - \mathcal{F}_{i-1/2}^n - \varepsilon_{i-1/2,R}^n\right] + \Delta t \mathcal{S}_i^n$$

- $\mathcal{F}_{i+1/2}^n$: conservative flux (pressure and convection)
- $\varepsilon_{i+1/2,L}^n$, $\varepsilon_{i+1/2,R}^n$: discontinuous bathymetry contributions
- $\mathcal{S}_i^n$: continuous bathymetry contributions

Hydrostatic reconstruction



Step 1: $b^n_{i+1/2} = \max(b^n_{i+1/2,L}, b^n_{i+1/2,R})$

Step 2: $h^{*,n}_{i+1/2,L} = \max(0, h^n_{i+1/2,L} - b^n_{i+1/2} + b^n_{i+1/2,L})$

Step 3: $\eta^{*,n}_{i+1/2,L} = h^{*,n}_{i+1/2,L} + b^n_{i+1/2}$

Step 4: $u^{*,n}_{i+1/2,L} = u^n_{i+1/2,L}$, $u^{*,n}_{i+1/2,R} = u^n_{i+1/2,R}$

Fluxes

$\mathcal{F}^n_{i-1/2} = \mathbb{F}(W^{*,n}_{i-1/2,L}, W^{*,n}_{i-1/2,R})$ with

$$\mathbb{F}((h,u),(h',u')) = \frac{1}{2}\begin{pmatrix} hu \\ hu^2 + \frac{gh^2}{2} \end{pmatrix} + \frac{1}{2}\begin{pmatrix} h'u' \\ h'u'^2 + \frac{gh'^2}{2} \end{pmatrix} - \lambda \begin{pmatrix} h - h' \\ hu - h'u' \end{pmatrix}$$

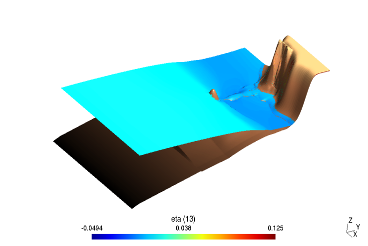
$$\varepsilon^n_{i+1/2,L} = \frac{g}{2}\left[(h^{*,n}_{i+1/2,L})^2 - (h^n_{i+1/2,L})^2\right]$$

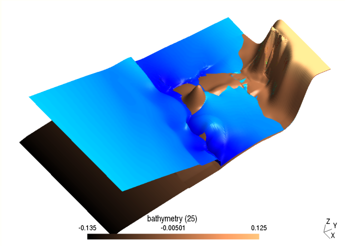
$$\mathcal{S}^n_i = -g\frac{h^n_{i+1/2,L} + h^n_{i-1/2,R}}{2} \times \frac{b^n_{i+1/2,L} - b^n_{i-1/2,R}}{\Delta x_i}$$

Numerical simulations

Monai

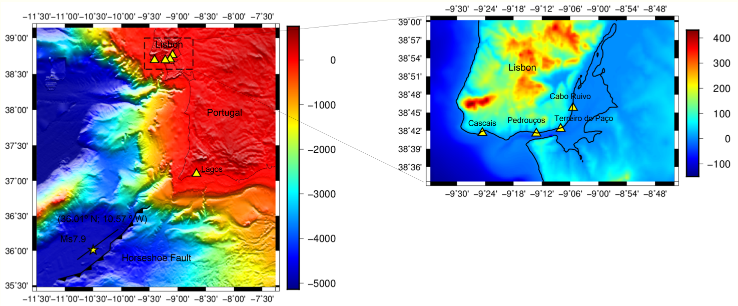
- Laboratory benchmark the extreme Monai run-up
- Consequence of the 1993 Okushiri tsunami





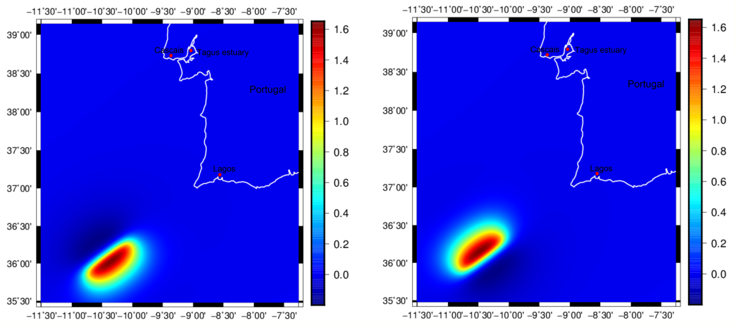
Tagus

The 28 February 1969 event was a submarine earthquake Ms7.9



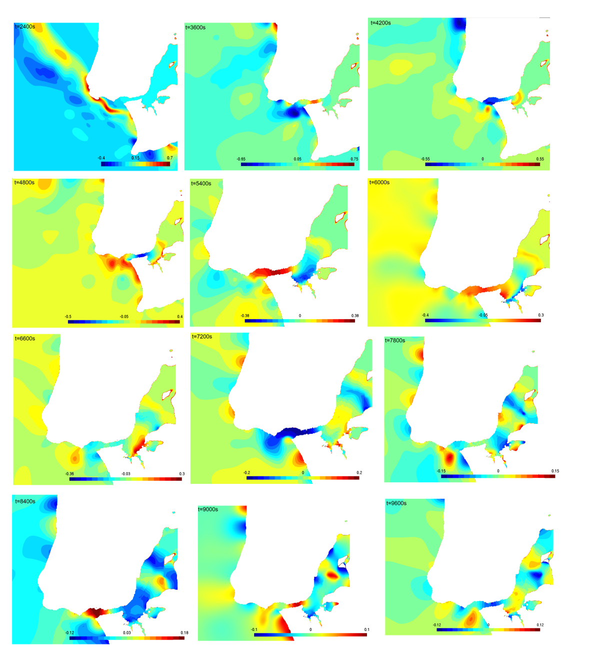
Records registered by the tide stations in five points

Two possible configurations (polarity) SW-NE fault vs NE-SW fault simulation

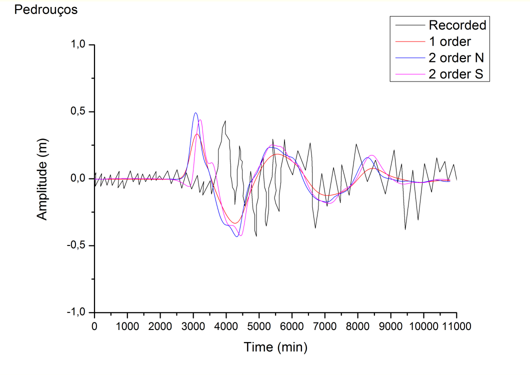


Initial water height: -0.9 to +1.6 meters

Propagation wave



measured vs simulation



- Almost same travelling time
- Recover the low frequency (big structures $> 10km$)
- Lost high frequency (small structures $< 1km$)

Conclusions

- Efficient tools for prediction and scenario analysis
- 200 x200 m grid need 6 hours of computation
- highly parallelizable algorithms with a lot of repetitive calculations
- need 10 x 10 m grids to catch human structures + beach
- new numerical methods developed at UM