#### Sistemas Digitais I LESI - 2° ano

#### Lesson 6 - Combinational Design Practices

Prof. João Miguel Fernandes
 (miguel@di.uminho.pt)

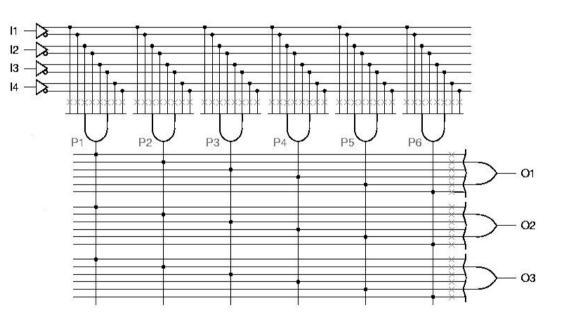
Dept. Informática



UNIVERSIDADE DO MINHO

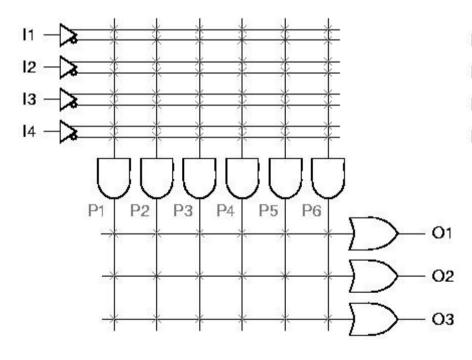
- The first PLDs were <u>Programmable Logic Arrays</u> (PLAs).
- A PLA is a combinational, 2-level AND-OR device that can be programmed to realise any sum-of-products logic expression.
- A PLA is limited by:
  - the number of inputs (n)
  - the number of outputs (m)
  - the number of product terms (p)
- We refer to an "n x m PLA with p product terms". Usually,  $p \ll 2^{n}$ .
- An n x m PLA with p product terms contains p 2n-input AND gates and m p-input OR gates.

- Each input is connected to a buffer that produces a true and a complemented version of the signal.
- Potential connections are indicated by Xs.
- The device is programmed by establishing the needed connections.
- The connections are made by fuses.

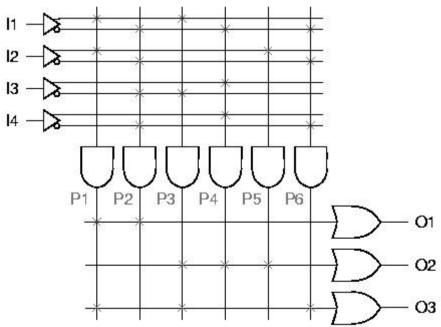


A 4x3 PLA with 6 product terms.

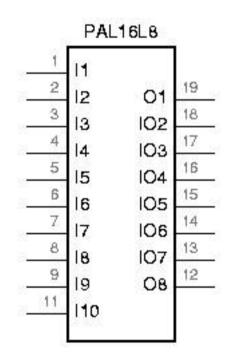
 Compact representation of the 4x3 PLA with 6 product terms.



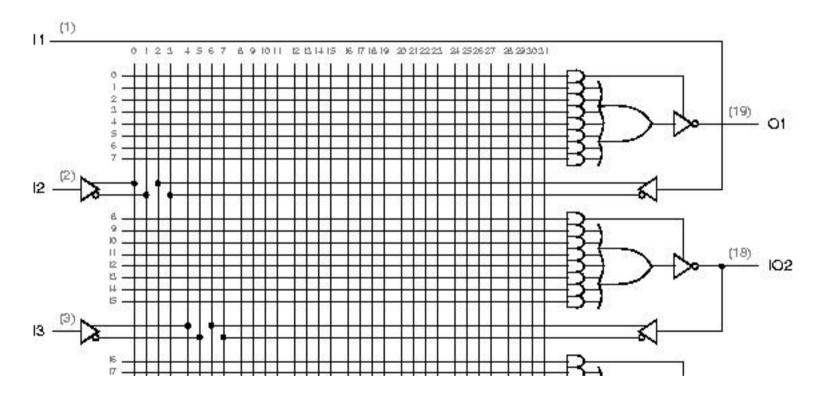
•  $O1 = |1 \cdot |2 + |1' \cdot |2' \cdot |3' \cdot |4'$   $O2 = |1 \cdot |3' + |1' \cdot |3 \cdot |4 + |2$  $O3 = |1 \cdot |2 + |1 \cdot |3' + |1' \cdot |2' \cdot |4'$ 



- Another PLD is PAL (Programmable Array Logic).
- A PAL device has a fixed OR array.
- In a PAL, product terms are not shared by the outputs.
- A PAL is usually faster than a similar PLA.



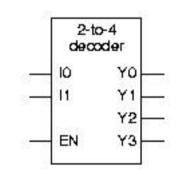
• Part of the logic diagram of the PAL 16L8.

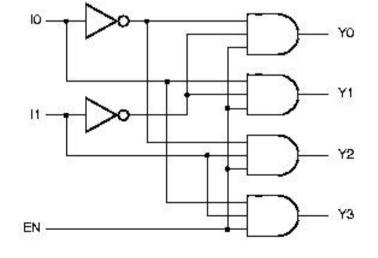


#### 6. Combinational Practices - Decoders (1) -

- A decoder is a circuit that converts coded inputs into coded outputs.
- Usually, the input code has fewer bits than the output code.
- The most common decoder is an n-to-2<sup>n</sup> or binary decoder.
- A binary decoder is used when one of 2<sup>n</sup> outputs needs to be activated based on an n-bit input value.

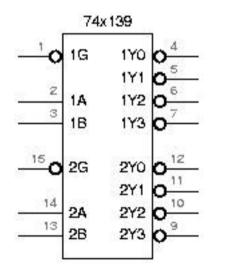
inputs			Outputs				
EN	11	10	YЭ	¥2	ΥI	YO	
0	x	x	0	0	0	0	
1	0	o	0	0	0	1	
1	0	1	0	0	1	0	
1	1	0	0	1	0	0	
1	1	1	1	0	0	0	





#### 6. Combinational Practices - Decoders (2) -

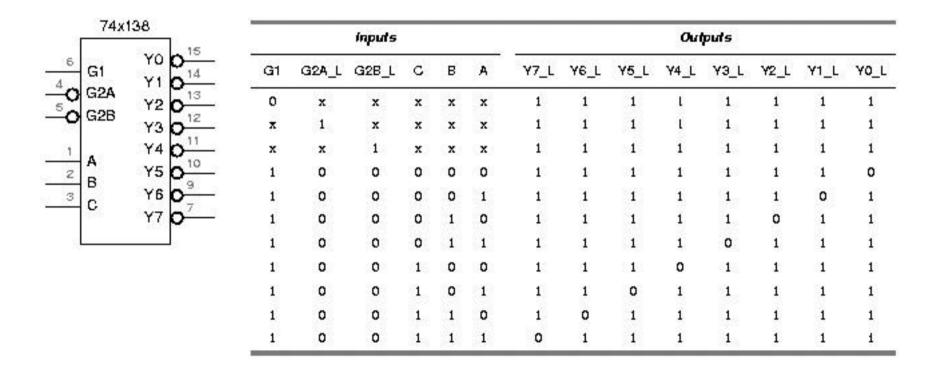
• A 74x139 IC has two independent 2-to-4 decoders.



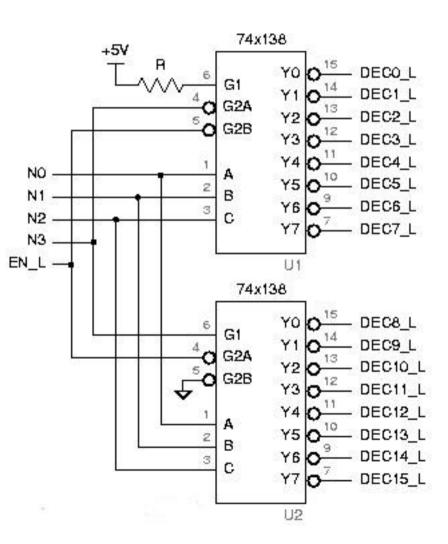
ín	puls		Outputs				
G_L	в	A	¥9_L	¥2_L	¥1_L	YO_L	
1	x	x	1	1	1	1	
0	0	0	1	1	1	0	
0	0	1	1	1	o	1	
0	1	0	1	0	1	1	
0	1	1	0	1	1	1	

#### 6. Combinational Practices - Decoders (3) -

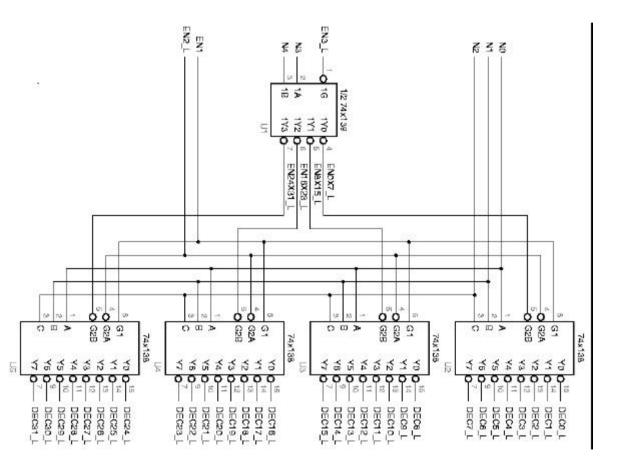
A 74x138 IC has one 3-to-8 decoder.



- Multiple decoders can be used to decode larger code words.
- The top decoder (U1) is enabled when N3 is 0, and the bottom decoder (U2) is enabled when N3 is 1.
- To handle larger code words, decoders can be cascaded hierarchically.



- To handle larger code words, decoders can be cascaded hierarchically.
- A 5-to-32 decoder can be built with one 2-to-4 and four 3-to-8 decoders.
- The 2-to-4 decoder treats the high order bits.
- The 3-to-8 decoders treat the low-order bits.



#### 6. Combinational Practices - Decoders (6) -

- There are several ways to write decoders in VHDL.
- The most primitive would be to write a structural description equivalent to the logic circuit on slide 7.

```
library IEEE;
use IEEE.std logic 1164.all;
entity V2to4dec is
 port (IO, I1, EN: in STD LOGIC;
        Y0, Y1, Y2, Y3: out STD LOGIC );
end V2to4dec;
Architecture V2to4dec s of V2to4dec is
  signal NOTIO, NOTI1: STD LOGIC;
  component inv port (I: in STD_LOGIC; O: out STD_LOGIC ); end component;
  component And3 port (I0, I1, I2: in STD LOGIC; O: out STD LOGIC ); end component;
begin
  U1: inv port map (10,NOTIO);
  U2: inv port map (I1, NOTI1);
 U3: and3 port map (NOTI0, NOTI1, EN, Y0);
  U4: and3 port map ( I0,NOTI1,EN,Y1);
 US: And3 port map (NOTIO, 11, EN, Y2);
  U6: and3 port map ( I0, I1, EN, Y3);
end V2to4dec s;
```

#### 6. Combinational Practices - Decoders (7) -

The second alternative is using the dataflow style.

```
library IEEE;
use IEEE.std logic 1164.all;
entity V74x138 is
    port (G1, G2A_L, G2B_L: in STD_LOGIC; -- enable inputs
        A: in STD LOGIC VECTOR (2 downto 0); -- select inputs
        Y L: out STD LOGIC VECTOR (0 to 7) ); -- decoded outputs
 end V74x138;
Architecture V74x138 & of V74x138 is
  signal Y_L_i: STD_LOGIC_VECTOR (0 to 7);
begin
  with A select Y_L_i <=
    "011111111" when "000",
    "101111111" when "001".
    "110111111" when "010",
    "111011111" when "011",
    "11110111" when "100",
    "11111011" when "101",
    "111111101" when "110".
    "111111110" when "111",
    "111111111" when others;
  Y_L <= Y_L_i when (G1 and not G2A_L and not G2B_L)='1' else "11111111";
end V74x138_a;
```

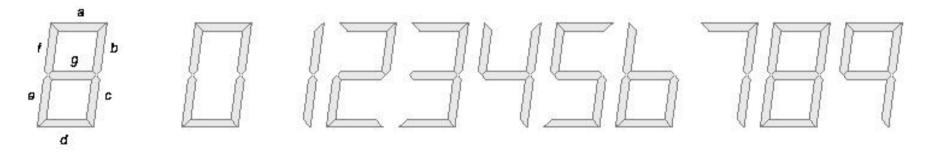
#### 6. Combinational Practices - Decoders (8) -

Another alternative is using the behavioral style.

```
architecture V3to8dec_c of V3to8dec is
begin
process (G1, G2, G3, A)
variable i: INTEGER range 0 to 7;
begin
Y <= "00000000";
if (G1 and G2 and G3) = '1' then
for i in 0 to 7 loop
if i=CONV_INTEGER(A) then Y(i) <= '1'; end if;
end loop;
end if;
end process;
end V3to8dec_c;
```

#### 6. Combinational Practices - 7-Segment Decoders (1) -

- A <u>7-segment display</u> is used in watches, calculators, and devices to show decimal data.
- A digit is displayed by illuminating a subset of the 7 line segments.



• A <u>7-segment decoder</u> has a 4-bit BCD as its input and the 7-segment code as its output.

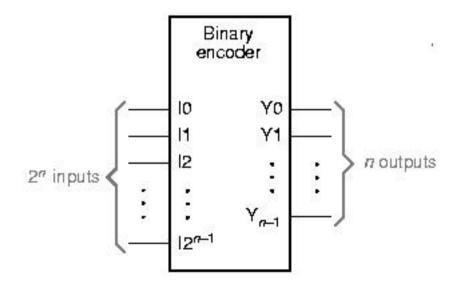
#### 6. Combinational Practices - 7-Segment Decoders (2) -

	ínputs					Outputs					
BI_L	D	С	в	А	а	ь	c	d	e	f	9
0	x	x	x	x	0	0	0	0	0	0	0
1	0	0	0	0	1	1	1	1	1	1	0
1	о	0	o	1	o	1	1	0	о	o	0
1	0	0	1	0	1	1	0	1	1	0	1
1	0	0	1	1	1	1	1	1	0	0	1
1	o	1	o	0	o	1	1	0	o	1	1
1	0	1	0	1	1	0	1	1	0	1	1
1	0	1	1	0	0	0	1	1	1	1	1
1	o	1	1	1	1	1	1	0	o	o	0
1	1	0	0	0	1	1	1	1	1	1	1
1	1	o	0	1	1	1	1	0	0	1	1
1	1	o	1	o	o	o	0	1	1	0	1
1	1	0	1	1	0	0	1	1	0	0	1
1	1	1	0	0	0	1	0	0	0	1	1
1	1	1	0	1	1	0	0	1	0	1	1
1	1	1	1	0	0	0	0	1	1	1	1
1	1	1	1	1	o	0	0	o	o	0	0

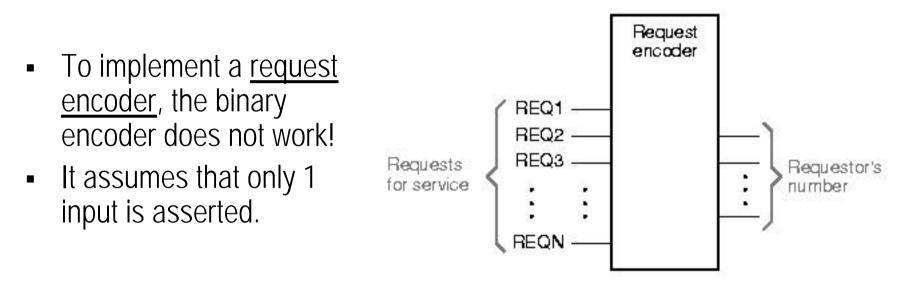
- Exercise 1: Obtain minimised expressions for outputs of the 7segment decoder.
- Exercise 2: Write a VHDL description of a 7segment decoder.

#### 6. Combinational Practices - Encoders (1) -

- An <u>encoder</u> is a circuit whose output code has normally fewer bits than its input code.
- The simplest encoder to build is a 2<sup>n</sup>-to-n or binary encoder. It has the opposite function as a binary encoder.
- Equations for an 8-to-3 encoder : Y0 = I1 + I3 + I5 + I7 Y1 = I2 + I3 + I6 + I7 Y2 = I4 + I5 + I6 + I7
- Only 1 input is active at a time.
   What happens if 2 inputs are asserted (ex: I2 and I4)?

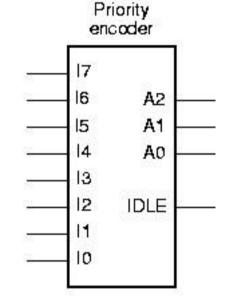


#### 6. Combinational Practices - Encoders (2) -



- If multiple requests can be made simultaneously, a <u>priority</u> must be assigned to the input lines.
- When multiple requests are made, the device (priority encoder) produces the number of the highest-priority requestor.

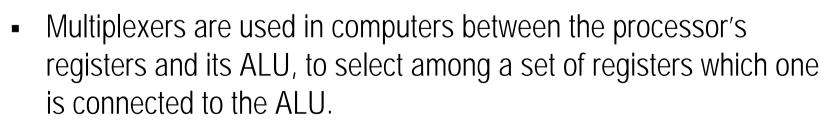
- Input I7 has the highest priority.
- Outputs A2-A0 contain the number of the highest-priority asserted input, if any.
- The IDLE output is asserted if no inputs are asserted.
- Intermediate variable Hi is 1, if Ii is the highest priority 1-input: H7 = I7 H6 = I6·I7' H5 = I5·I6'·I7' H4 = I4·I5'·I6'·I7' ... (similar equations for H3-H0)

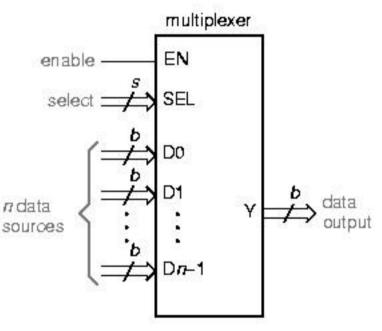


- A0 = H1 + H3 + H5 + H7
  A1 = H2 + H3 + H6 + H7
  A2 = H4 + H5 + H6 + H7
- IDLE= 10'·11'·12'·13'·14'·15'·16'·17'

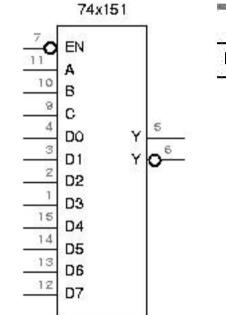
#### 6. Combinational Practices - Multiplexers (1) -

- A <u>multiplexer</u> (mux) is a digital switch.
- It connects data from one of n sources to its output.
- The SEL input selects among the n sources, so s = [log<sub>2</sub> n].
- When EN=0, Y=0;
   When EN=1, the mux is working.





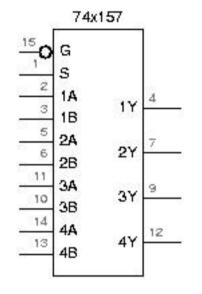
- A 74x151 IC has one 8input, 1-bit multiplexer.
- The select inputs are named A,B,C, where C is the MSB.
- The enable input EN\_L is active low.
- Both active-low and high versions of the output are provided



	Inpi	Oa	tputs 🛛		
EN_L	C	В	A	Ŷ	Y_L
1	X	×	×	0	1
0	0	0	0	D0	D0'
0	0	0	1	D1	D1'
0	0	1	0	D2	D2′
0	0	1	÷.	D3	D3'
0	10	0	0	D4	D4'
0	10	0	1	DS	D5'
0	1	1	0	D6	D6'
0	1	1	1	D7	D7'

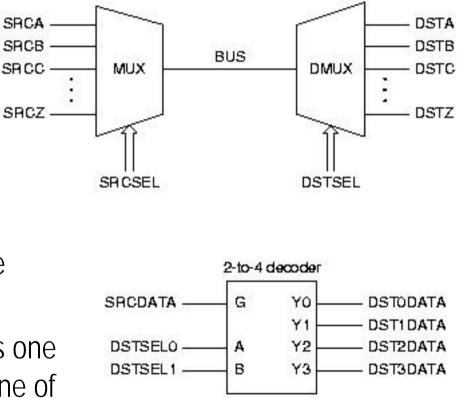
- Multiplexers (3) -

- A 74x157 IC has one 2-input, 4-bit multiplexer.
- The select input is S.
- The enable input G\_L is active low.
- The truth table was extended and inputs appear at the outputs columns.



Inputs		Outputs				
G_L	S	14	2Y	ЗY	4Y	
1	X	0	0	0	0	
0	0	1A	2A	ЗA	4A	
0	1	16	28	36	46	
and in the second	100					

- A multiplexer can be used to select one of n sources of data to transmit on a bus.
- At the other end, a demultiplexer can be used to route the bus to one of m destinations.
- The function of a multiplexer is the inverse of a demultiplexer's.
- A 1-bit, n-output demultiplexer has one data input and s inputs to select one of the n=2<sup>s</sup> data outputs.



#### 6. Combinational Practices - Multiplexers (5) -

- It is easy to describe multiplexers in VHDL.
- In the dataflow style, a SELECT statement is required.

```
library IEEE;
use IEEE. std_logic_1104.all;
entity mux4inBb is
    port (
        S: in STD_LOGIC_VECTOR (1 downto 0);
                                                -- Select imputs, 0-3 ==> A-D
        A, B, C, D: in STD_LOGIC_VECTOR (1 to B); -- Data bus input
        Y: out STD_LOGIC_VECTOR (1 to B)
                                              -- Data bus output
    33.
and mussings;
architecture mundiage of mundiage is
begin
    with 5 select Y <=
     A when "00",
     B when "OI".
     C when "10",
     D when "II".
     (others =: 'U') when others: -- this creates an B-bit vector of 'U'
and mussings;
```

#### 6. Combinational Practices - Multiplexers (6) -

• In a behavioural architecture, a CASE statement is used.

```
architecture mux&inEp of mux&inEb is
begin
process(S, A, B, C, D)
begin
case 5 is
when *00* => Y <= A;
when *01* => Y <= B;
when *10* => Y <= C;
when *10* => Y <= D;
when others => Y <= (others => 'U'); -- B-bit vector of 'U'
end case;
end process;
end mux&inEp;
```

 It is easy to customise the selection criteria in a VHDL multiplexer program.

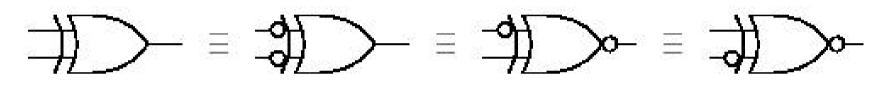
- XOR and Parity Circuits (1) -

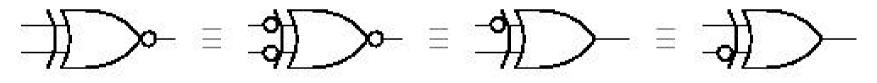
- An <u>Exclusive-OR (XOR)</u> gate is a 2-input gate whose output is 1, if exactly one of its inputs is 1.
- An XOR gate produces a 1 output if its input are different.

x	Y	X⊕ Y (XOR)	(X⊕Y) (XNOR)	
0	0	0	1	
0	1	1	0	
S <b>1</b> 8	0	12	. <b>O</b>	
<b>1</b>	1	0	<b>1</b>	

- An <u>Exclusive-NOR (XNOR)</u> is just the opposite: it produces a 1 output if its inputs are the same.
- The XOR operation is denoted by the symbol  $\oplus$ .
- $X \bigoplus Y = X' \cdot Y + X \cdot Y'$

• There are 4 symbols for each XOR and XNOR function.

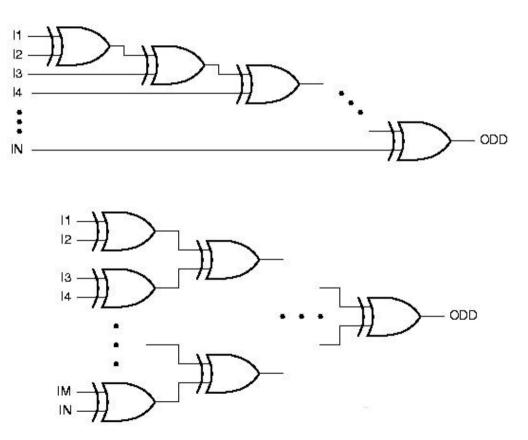




- These alternatives are a consequence of the following rule:
  - Any two signals (inputs or output) of an XOR or XNOR gate may be complemented without changing the resulting logic function.
- In bubble-to-bubble design we choose the symbol that is most expressive of the logic function being performed.

- XOR and Parity Circuits (3) -

- n XOR gates may be cascaded to form a circuit with n+1 inputs and a single output. This is a <u>odd-parity circuit</u>, because its output is 1 if an odd number of its inputs are 1.
- If the output of either circuit is inverted, we get an even-parity circuit, whose output is 1 if an even number of its inputs are 1.



- XOR and Parity Circuits (4) -

- VHDL provides the primitive operators xor and xnor.
- A 3-input XOR device can be specified in VHDL dataflow style program.

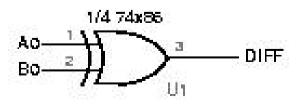
```
library IEEE;
use IEEE.std_logic_1104.all;
entity vmor3 is
    port (A, B, C: in STD_LOGIC)
        Y: out STD_LOGIC);
end vmor3;
architecture vmor3 of vmor3 is
begin
    Y <= A mor B mor C;
end vmor3;
```

- XOR and Parity Circuits (5) -

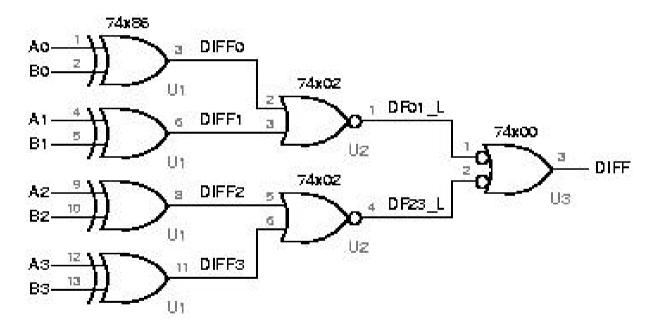
 A 9-input parity function can be specified behaviourally.

```
library IEEE;
use IEEE.std_logic_1104.all;
entity parity9 is
 port (I: in STD_LOGIC_VECTOR (1 to 9);
        EVEN, CDD: out STD_LOGIC);
end parity9;
architecture parity9p of parity9 is
begin
process (I)
  variable p : STD_LOGIC;
 Ърділ
   p := I(1);
    for j in Z to 9 loop
      if I(j) = ii then p := not p; and if;
   end loop;
    ODD <= p;
    EVEN <= not p;
 and process;
end parity9p;
```

- Comparing two binary words is a common operation in computers.
- A circuit that compares 2 binary words and indicates whether they are equal is a <u>comparator</u>.
- Some comparators interpret their input as signed or unsigned numbers and also indicate an arithmetic relationship (greater or less than) between the words.
- These circuits are often called <u>magnitude comparators</u>.
- XOR and XNOR gates can be viewed as 1-bit comparators.
- The DIFF output is asserted if the inputs are different.



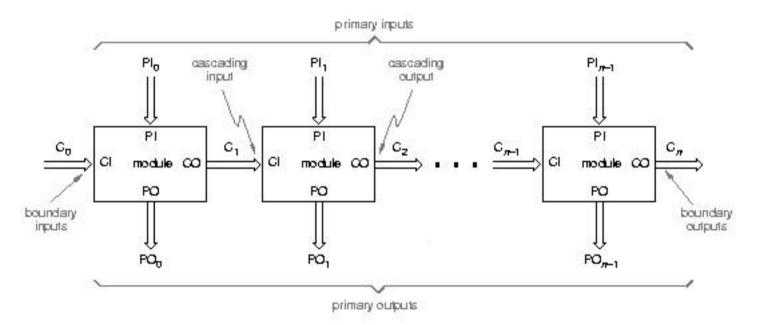
• The outputs of 4 XOR gates can be ORed to create a 4-bit comparator.



- The DIFF output is asserted if any of the input-bit pairs are different.
- This circuit can be easily adapted to any number of bits per word.

#### 6. Combinational Practices - Comparators (3) -

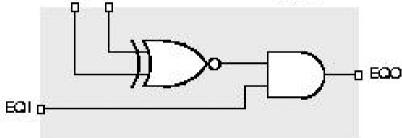
• An <u>iterative circuit</u> is a combinational circuit with the following structure.



- The circuit contains n identical modules, each of which has <u>both primary</u> <u>inputs and outputs</u> and <u>cascading inputs and outputs</u>.
- The left-most cascading inputs are usually connected to fixed values.

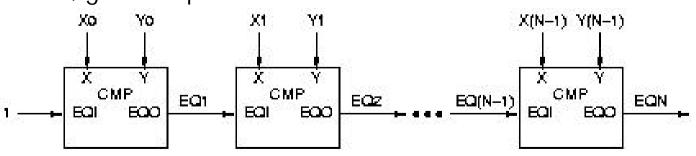
#### 6. Combinational Practices - Comparators (4) -

- Two n-bit values X and Y can be compared one bit at a time using a single bit EQ<sub>i</sub> at each step to keep track of whether all of the bit-pairs have been equal so far:
- 1. Set EQ<sub>0</sub> to 1 and set *i* to 0.
   2. If EQ<sub>i</sub> is 1 and X<sub>i</sub>=Y<sub>i</sub>, set EQ<sub>i+1</sub> to 1. Else set EQ<sub>i+1</sub> to 0.
  - 3. Increment *i*.

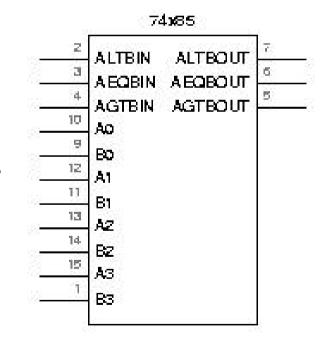


CMP.

4. If *i* < *n*, go to step 2.

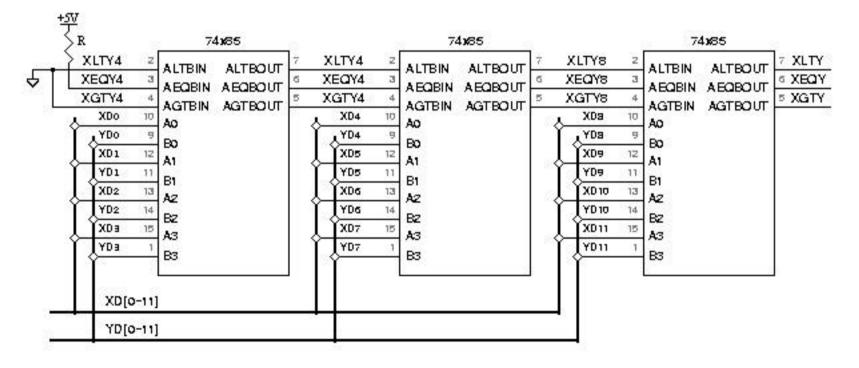


- Several MSI comparators have been developed commercially.
- The 74x85 is a 4-bit comparator.
- It provides a greater-than output, a less-than output and an equal output.
- The 74x85 also has cascading inputs for combining multiple chips to create comparators for more than 4 bits.
- AGTBOUT =  $(A>B) + (A=B) \cdot AGTBIN$ AEQBOUT =  $(A=B) \cdot AEQBIN$ ALTBOUT =  $(A<B) + (A=B) \cdot ALTBIN$



#### 6. Combinational Practices - Comparators (6) -

• With three 74x85 circuits, a 12-bit comparator can be built.



- VHDL has comparison operators for all of its built-in types.
- Equality (=) and inequality (/=) operators apply to all types.
- For array and record types, the operands must have equal size and structure, and the operands are compared component by component.
- VHDL's other comparison operators (>, <, >=, <=) apply only to integers, enumerated types and one-dimensional arrays of enumeration or integer types.

- Adders, Subtractors and ALUs (1) -

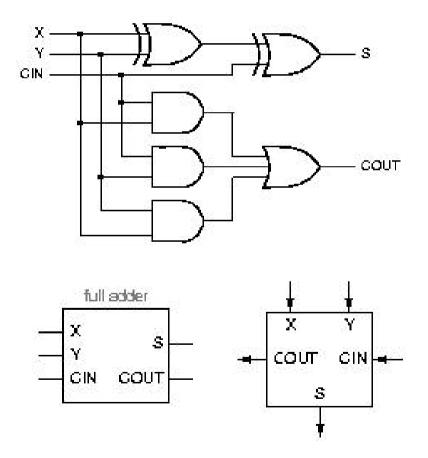
- Addition is the most commonly performed arithmetic operation in digital systems.
- An <u>adder</u> combines two arithmetic operands using the addition rules.
- The same addition rules, and hence the same adders, are used for both unsigned and 2's complement numbers.
- An adder can perform subtraction as the addition of the minuend and the complemented subtrahend.
- A <u>subtractor</u> can also be built to perform subtraction directly.
- An <u>ALU</u> (Arithmetic and Logic Unit) performs addition, subtraction, and other logical operations.

- Adders, Subtractors and ALUs (2) -

- The simplest adder, called a <u>half adder</u>, adds two 1-bit operands X and Y, producing a 2-bit sum.
- The sum can range from 0 to 2, which requires two bits to express.
- The low-order bit of the sum may be named HS (half sum).
- The high-order bit of the sum may be named CO (carry out).
- The following equations can be written:
   HS = X ⊕ Y = X · Y' + X' · Y
   CO = X · Y
- To add operands with more than one bit, carries between bit positions must be provided.

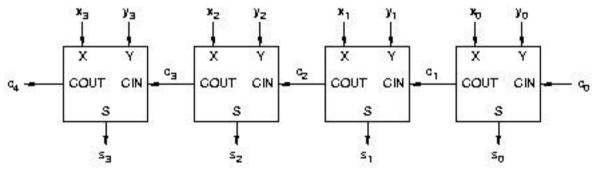
- Adders, Subtractors and ALUs (3) -

- The building block for this operation is called a <u>full adder</u>.
- Besides the addend-bit inputs X and Y, a full adder has a <u>carry-bit input</u>, CIN.
- The sum of the 3 bits can range from 0 to 3, which can still be expressed with just two output bits, S and COUT.
- The following equations can be written:
   S = X ⊕ Y ⊕ CIN
   COUT = X·Y + X·CIN + Y·CIN



- Adders, Subtractors and ALUs (4) -

- Two binary words, each with n bits, can be added using a ripple adder.
- A <u>ripple adder</u> is a cascade of n full-adders stages, each of which handles one bit.



- The carry input to the least significant bit  $(c_0)$  is usually set to 0.
- The carry output of each full adder is connected to the carry input of the next most significant full adder.

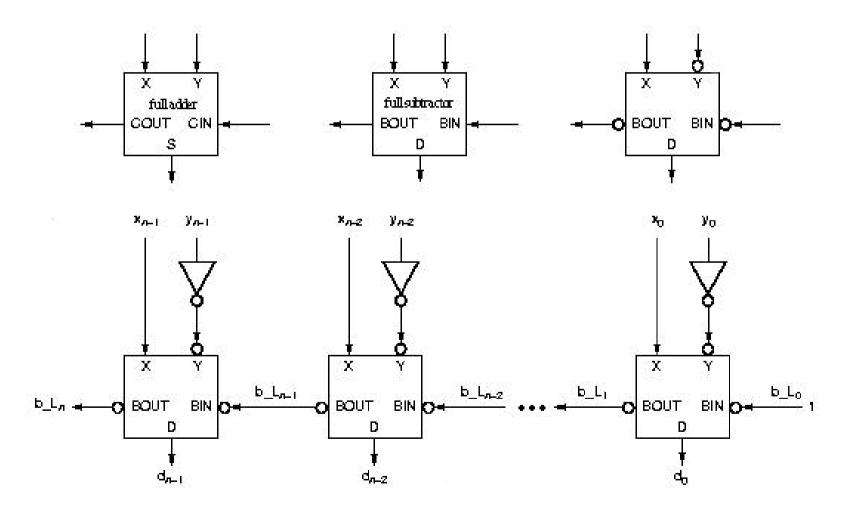
- Adders, Subtractors and ALUs (5) -

- The binary subtraction operation is analogous to binary addition.
- A <u>full subtractor</u> has inputs X (minuend), Y (subtrahend) and BIN (borrow in) and outputs D (difference) and BOUT (borrow out).
- The following equations can be written:

 $D = X \bigoplus Y \bigoplus BIN$  $BOUT = X' \cdot Y + X' \cdot BIN + Y \cdot BIN$ 

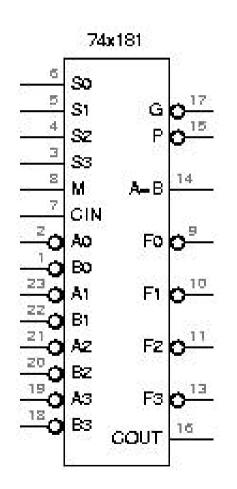
- These equations are similar to the equations for a full adder.
   D = X ⊕ Y' ⊕ BIN'
   BOUT = X·Y' + X·BIN' + Y'·BIN'
- A full subtractor can be built from a full adder. X-Y = X+Y'+1

- Adders, Subtractors and ALUs (6) -



- Adders, Subtractors and ALUs (7) -

- An <u>ALU</u> is a combinational circuit that can perform several arithmetic and logical operations on a pair of b-bit operands.
- The operation to be performed is specified by a set of function-select inputs.
- Typical MSI ALUs have 4-bit operands and three to five function-select inputs, allowing up to 32 different functions to be performed.
- A 74x181 IC has one 4-bit ALU.
- The operation performed by the 74x181 is selected by the M and S3-S0 inputs.



- Adders, Subtractors and ALUs (8) -

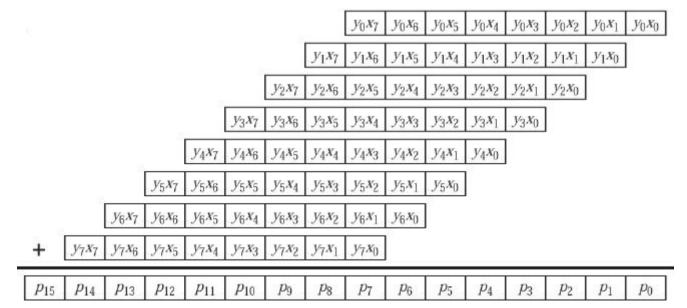
top ats				Function	
S3	S2	S1	<b>S</b> 0	M=0 (arithmetic)	.M = 1 (logic)
0	0	30.0	0	F= A minus 1 plus CIN	F = A'
0	0	3 <b>9</b> 3	<u>_1</u>	F = A · B minus 1 plus CIN	F = A' + B'
0	0	1	0	F= A · B' minus 1 plusCIN	F = A' + B
0	0	1	1	F= 1111 plus CIN	F = 1111
0	1	0	0	F = A plus (A + B') plus CIN	$F=A'\cdot B'$
0	1	0	1	$F = A \cdot B p lus (A + B') p lus CIN$	F = 8'
0	1	1	0	F= A minus B minus 1 plus CIN	F = A 🕀 B'
00	1	18	39 <b>1</b>	F= A + B' plus CIN	F = A + B'
1	0	30 <b>0</b> 3	0	F = A plus(A + B) plus CIN	$F=A'\cdotB$
1	0	<u>0</u>	1	F= A plus B plus CIN	F = A ⊕ B
1	0	1	0	$F = A \cdot B'$ plus ( $A + B$ ) plus CIN	F = B
1	ø	1	1	F= A + B plus CIN	F = A + B
1	1	0	Q	F= A plus A plus CIN	F=0000
1	i.	Q	1	F= A · B plus A plus CIN	$F=A\cdotB'$
1	1	1	0	F = A ⋅ B′ plus A plus CIN	$\mathbf{F} = \mathbf{A} \cdot \mathbf{B}$
1	<u>:1</u>	13	9: <b>1</b>	F= A plus CIN	E = A

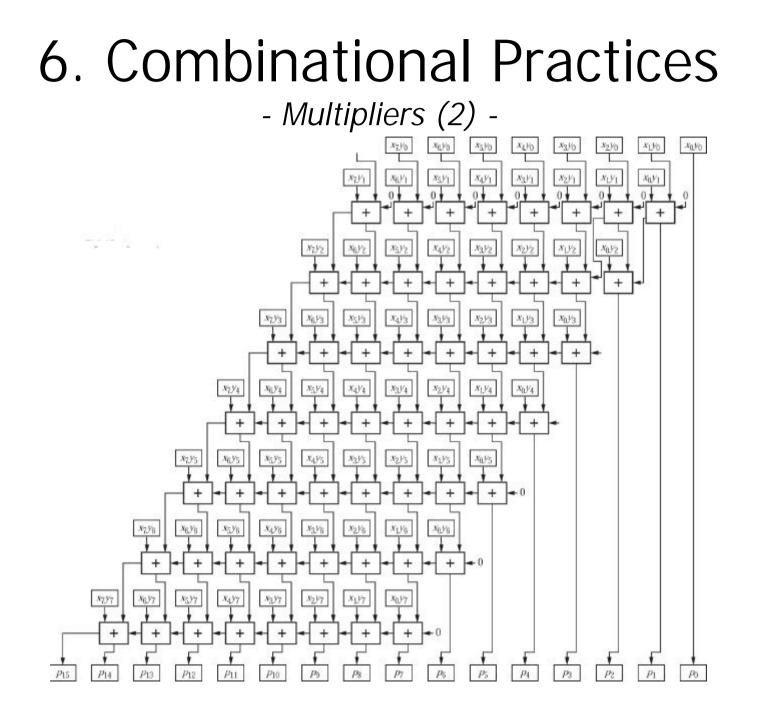
- Adders, Subtractors and ALUs (9) -

```
library IEEE;
use IEEE.std_logic_1164.all;
use IEEE.std_logic_arith.all;
entity vaddshr is
    port (
        A, B, C, D: in SIGNED (7 downto 0);
        SEL: in STD_LOGIC;
        S: out SIGNED (7 downto 0)
    );
end vaddshr;
architecture vaddshr_arch of vaddshr is
begin
        S <= A + B when SEL = '1' else C + D;
end vaddshr_arch;
```

- Multipliers (1) -

- The traditional algorithm to multiply binary numbers uses shifts and adds to obtain the result.
- However, it is not the only solution to implement a multiplier.
- Given 2 n-bit inputs (X, Y), we can write a truth table that expresses the 2n-bit product P=X×Y as a combinational function of X and Y.
- Most approaches to combinational multipliers are based on the traditional shift-and-add algorithm.





#### 6. Combinational Practices - Multipliers (3) -

```
library IEEE;
use IEEE.std_logic_1164.all;
use IEEE.std_logic_arith.all;
entity vmul8x8i is
    port (
        X: in UNSIGNED (7 downto 0);
        Y: in UNSIGNED (7 downto 0);
        P: out UNSIGNED (15 downto 0)
    );
end vmul8x8i;
architecture vmul8x8i_arch of vmul8x8i is
begin
    P <= X * Y;
end vmul8x8i_arch;
```