



# Master in Informatics Eng.

2015/16  
A.J.Proença

## The Roofline Performance Model (most slides are borrowed)

## Goals of the Roofline Model

CONVENTIONAL WISDOM in computer architecture produced similar designs. Nearly every desktop and server computer uses caches, pipelining, superscalar instruction issue, and out-of-order execution. Although the instruction sets varied, the microprocessors were all from the same school of

### Roofline Model

For the foreseeable future, off-chip memory bandwidth will often be the constraining resource in system performance.<sup>23</sup> Hence, we want a model that relates processor performance to off-chip memory traffic. Toward this

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**The Roofline model offers insight on how to improve the performance of software and hardware.**

BY SAMUEL WILLIAMS, ANDREW WATERMAN, AND DAVID PATTERSON

# Roofline:



- ❖ Multicore guarantees neither good scalability nor good (attained) performance
- ❖ Performance and scalability can be extremely non-intuitive even to computer scientists
- ❖ Success of the multicore paradigm seems to be premised upon their programmability
- ❖ To that end, one must understand the limits to both scalability and efficiency.

- How can we empower programmers?

**The Roofline Model:**  
A pedagogical tool for program analysis and optimization

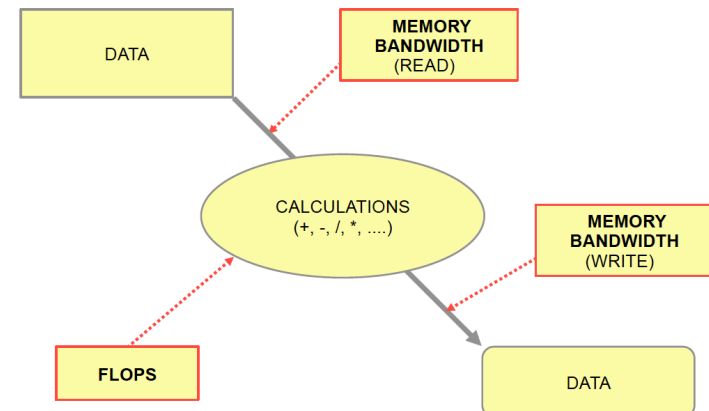
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## Performance Limiting Factors



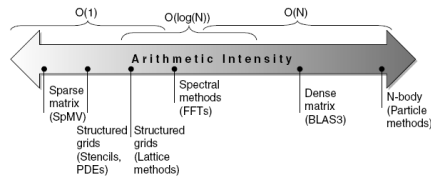
Leopold Grinberg  
IBM, T.J. Watson Research Center, USA

ICSC 2014, Shanghai, China



# Roofline Performance Model

- Basic idea:
  - Plot peak floating-point throughput as a function of arithmetic intensity
  - Ties together floating-point performance and memory performance for a target machine
- Arithmetic intensity
  - Floating-point operations per byte read



- ❖ There are three principal components to performance:
  - Computation
  - Communication
  - Locality
- ❖ Each architecture has a different balance between these
- ❖ Each kernel has a different balance between these
- ❖ Performance is a question of how well an kernel's characteristics map to an architecture's characteristics

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- ❖ For us, floating point performance (**Gflop/s**) is the metric of interest (typically double precision) ... but we could also consider SP or int
- ❖ Peak in-core performance can only be attained if:
  - fully exploit ILP, DLP, FMA, etc...
  - non-FP instructions don't sap instruction bandwidth
  - threads don't diverge (GPUs)
  - transcendental/non pipelined instructions are used sparingly
  - branch mispredictions are rare
- ❖ To exploit a form of in-core parallelism, it must be:
  - Inherent in the algorithm
  - Expressed in the high level implementation
  - Explicit in the generated code

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- ❖ For us, DRAM bandwidth (**GB/s**) is the metric of interest
- ❖ Peak bandwidth can only be attained if certain optimizations are employed:
  - Few unit stride streams
  - NUMA allocation and usage
  - SW Prefetching
  - Memory Coalescing (GPU)

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- ❖ Computation is free, Communication is expensive.
- ❖ Maximize locality to minimize communication
- ❖ **There is a lower limit to communication: compulsory traffic**
  
- ❖ Hardware changes can help minimize communication
  - Larger cache capacities minimize capacity misses
  - Higher cache associativities minimize conflict misses
  - Non-allocating caches minimize compulsory traffic
  
- ❖ Software optimization can also help minimize communication
  - Padding avoids conflict misses
  - Blocking avoids capacity misses
  - Non-allocating stores minimize compulsory traffic

3Cs model  
for caches

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## Preliminary notes in the Roofline Model

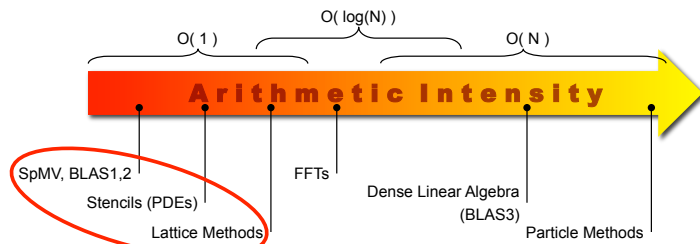
- goal: integrate in-core performance, memory bandwidth, and locality into a single readily understandable performance figure
- graphically show the penalty associated with not including certain software optimizations
- Roofline model will be unique to each architecture

- ❖ Temporal Locality
  - reusing data (either registers or cache lines) multiple times
  - amortizes the impact of limited bandwidth.
  - **transform loops or algorithms to maximize reuse.**
  
- ❖ Spatial Locality
  - data is transferred from cache to registers in words.
  - However, data is transferred to the cache in 64-128Byte lines
  - using every word in a line maximizes spatial locality.
  - **transform data structures into *structure of arrays* (SoA) layout**
  
- ❖ Sequential Locality
  - Many memory address patterns access cache lines sequentially.
  - CPU's hardware stream prefetchers exploit this observation to hide speculatively load data to memory latency.
  - **Transform loops to generate (a few) long, unit-stride accesses.**

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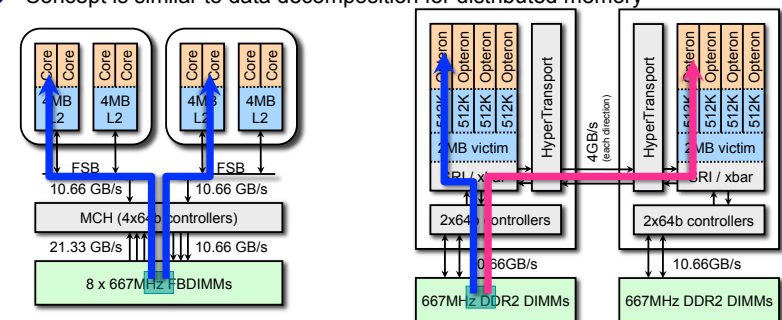
## Key elements in the Roofline Model

- x-axis: the “operational intensity”, operations per byte of RAM traffic, Flops/byte (traffic between caches and memory)
- y-axis: the attainable floating-point performance, GFlops/sec includes both peak processor/memory performance
- peak processor FP performance: a horizontal line computed from the processor specs
- peak memory performance: bounds the max FP performance of the memory system for a given operational intensity
- for each kernel: its performance is a point on a vertical line that crosses the x-axis on the kernel operational intensity



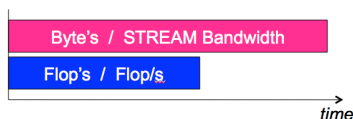
- ❖ True Arithmetic Intensity (AI) ~ Total Flops / Total DRAM Bytes
- ❖ Some HPC kernels have an arithmetic intensity that scales with problem size (increased temporal locality)
- ❖ Others have constant intensity
- ❖ Arithmetic intensity is ultimately limited by compulsory traffic
- ❖ Arithmetic intensity is diminished by conflict or capacity misses.

- ❖ Recent multicore SMPs have integrated the memory controllers on chip.
- ❖ As a result, memory-access is non-uniform (NUMA)
- ❖ That is, the bandwidth to read a given address varies dramatically among between cores
- ❖ Exploit NUMA (affinity+first touch) when you malloc/init data.
- ❖ Concept is similar to data decomposition for distributed memory



**Additional notes**

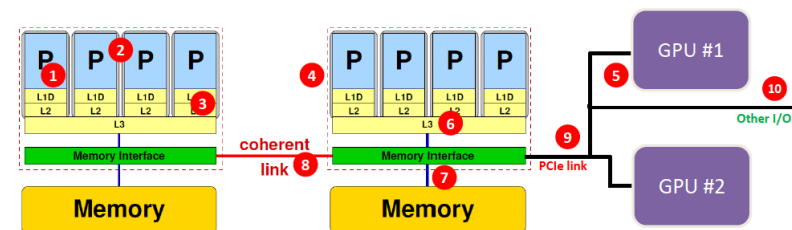
- Memory bandwidth #'s collected via micro benchmarks (or the STREAM benchmark)
- Computation #'s derived from optimization manuals (pencil and paper)
- Assume complete overlap of either communication or computation =>
 
$$\text{Gflop/s} = \min \begin{cases} \text{Peak Gflop/s} \\ \text{Stream BW} * \text{actual flop:byte ratio} \end{cases}$$



**Parallelism in a modern compute node**



- Parallel and shared resources within a shared-memory node



**Parallel resources:**

- Execution/SIMD units 1
- Cores 2
- Inner cache levels 3
- Sockets / ccNUMA domains 4
- Multiple accelerators 5

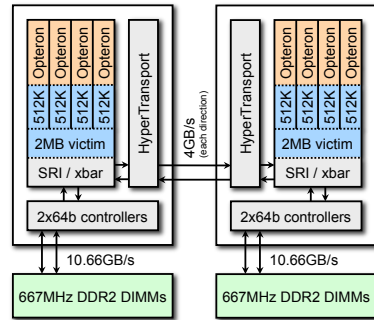
**Shared resources ("bottlenecks"):**

- Outer cache level per socket 6
- Memory bus per socket 7
- Intersocket link 8
- PCIe bus(es) 9
- Other I/O resources 10

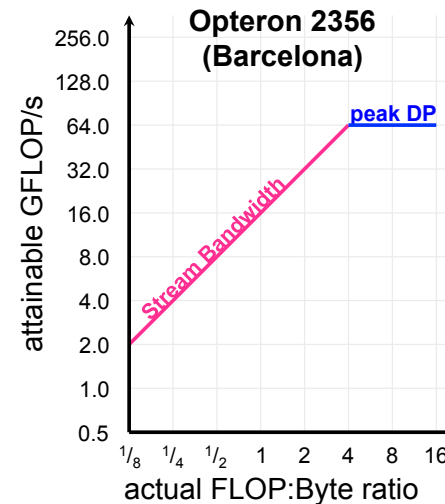
**Where is the bottleneck for your application?**

Basics of performance modeling for numerical applications: Roofline model and beyond

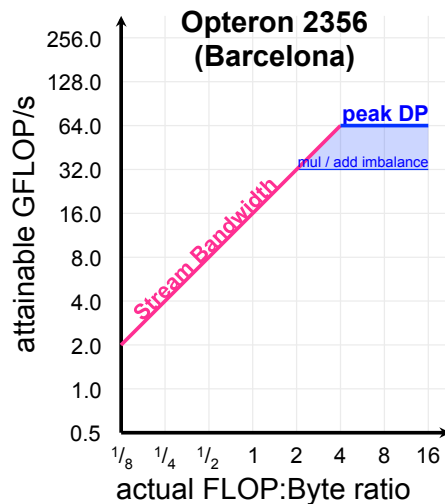
- ❖ Consider the Opteron 2356:
  - Dual Socket (NUMA)
  - limited HW stream prefetchers
  - quad-core (8 total)
  - 2.3GHz
  - 2-way SIMD (DP)
  - separate FPMUL and FPADD datapaths
  - 4-cycle FP latency



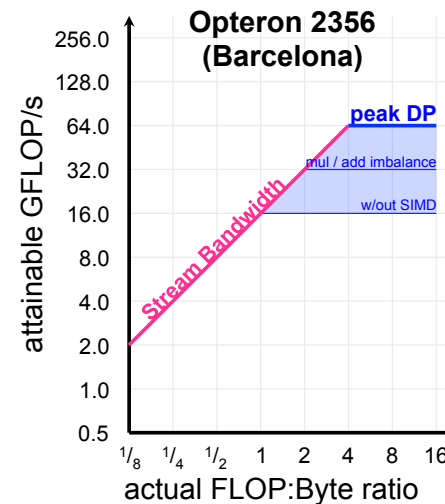
- ❖ Assuming **expression of parallelism** is the challenge on this architecture, what would the roofline model look like ?



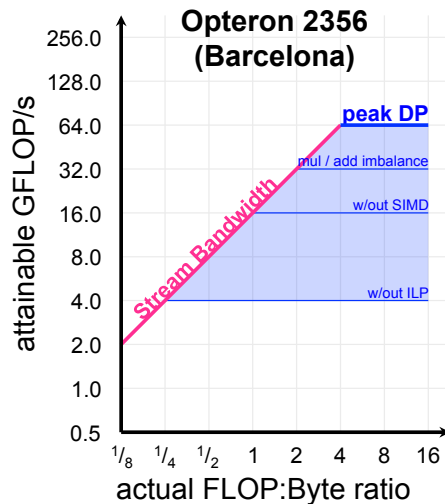
- ❖ Plot on log-log scale
- ❖ Given AI, we can easily bound performance
- ❖ But architectures are much more complicated
- ❖ We will bound performance as we eliminate specific forms of in-core parallelism



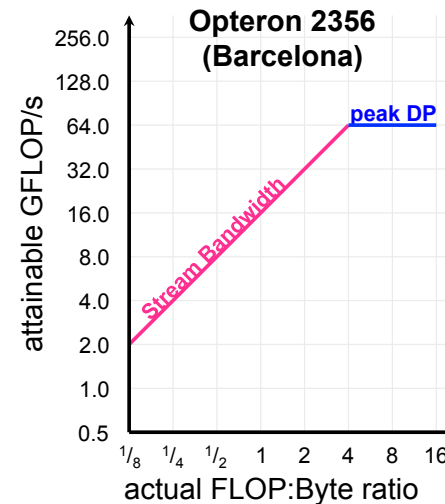
- ❖ Opterons have dedicated multipliers and adders.
- ❖ If the code is dominated by adds, then attainable performance is half of peak.
- ❖ We call these **Ceilings**
- ❖ They act like constraints on performance



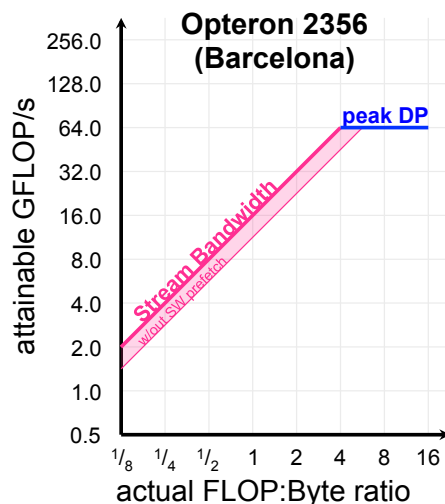
- ❖ Opterons have 128-bit datapaths.
- ❖ If instructions aren't SIMDized, attainable performance will be halved



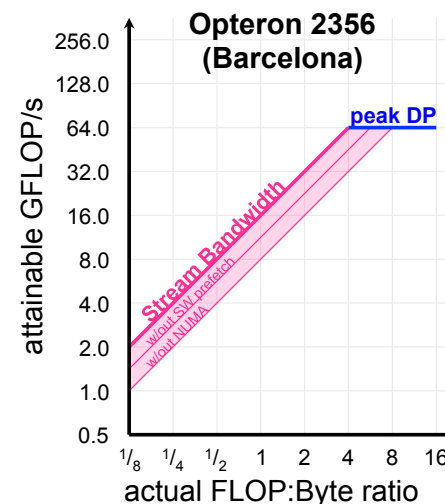
- ❖ On Opterons, floating-point instructions have a 4 cycle latency.
- ❖ If we don't express 4-way ILP, performance will drop by as much as 4x



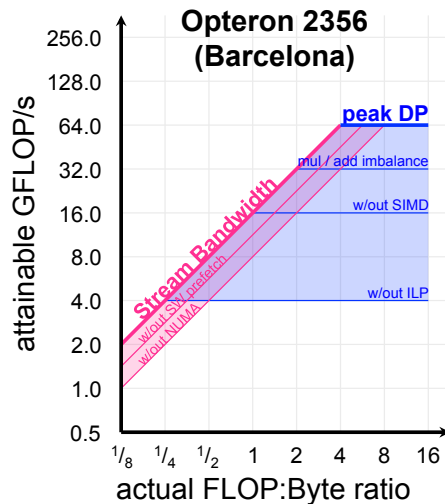
- ❖ We can perform a similar exercise taking away parallelism from the memory subsystem



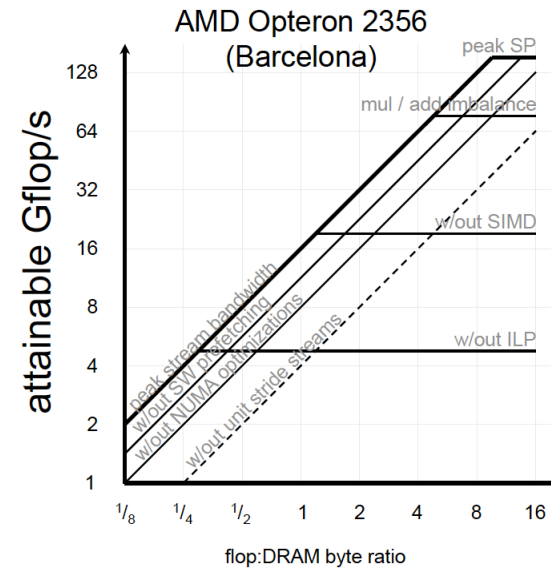
- ❖ Explicit software prefetch instructions are required to achieve peak bandwidth



- ❖ Opterons are NUMA
- ❖ As such memory traffic must be correctly balanced among the two sockets to achieve good Stream bandwidth.
- ❖ We could continue this by examining strided or random memory access patterns



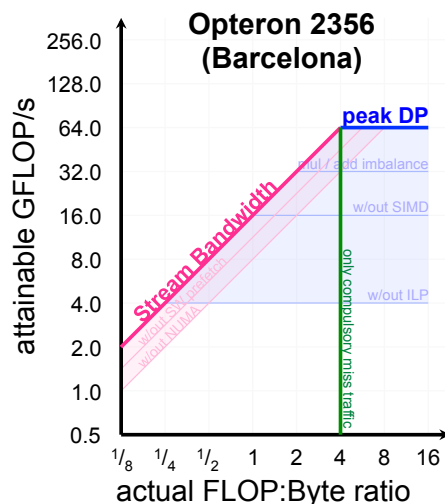
- ❖ We may bound performance based on the combination of expressed in-core parallelism and attained bandwidth.



- ❖ Bandwidth is much lower without unit stride streams

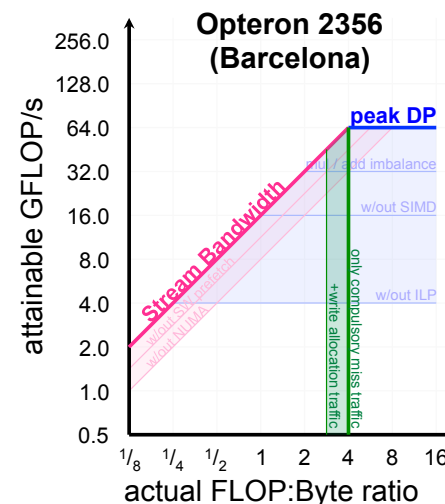
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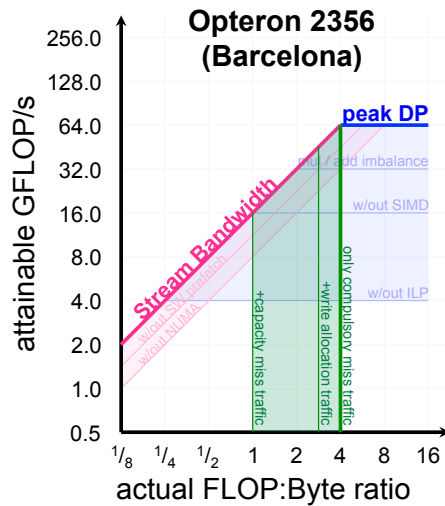
- ❖ Remember, memory traffic includes more than just compulsory misses.
- ❖ As such, actual arithmetic intensity may be substantially lower.
- ❖ Walls are unique to the architecture-kernel combination

$$AI = \frac{\text{FLOPs}}{\text{Compulsory Misses}}$$



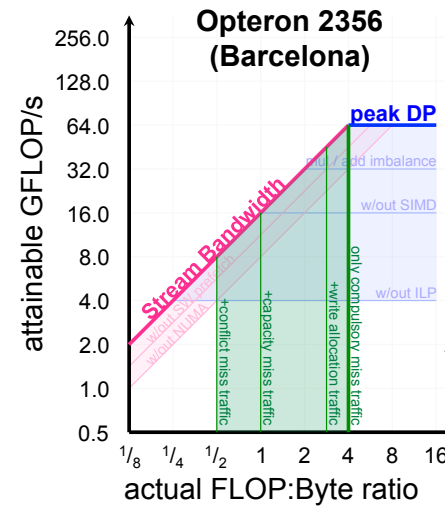
- ❖ Remember, memory traffic includes more than just compulsory misses.
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$$AI = \frac{\text{FLOPs}}{\text{Allocations} + \text{Compulsory Misses}}$$



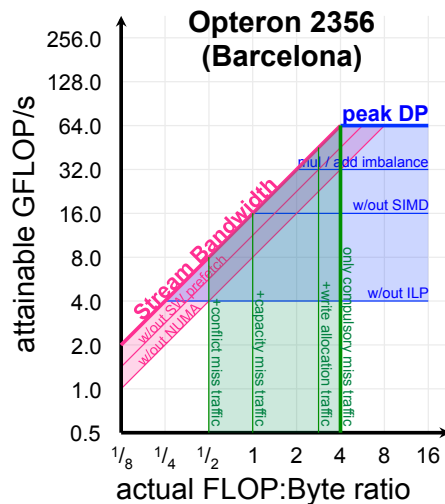
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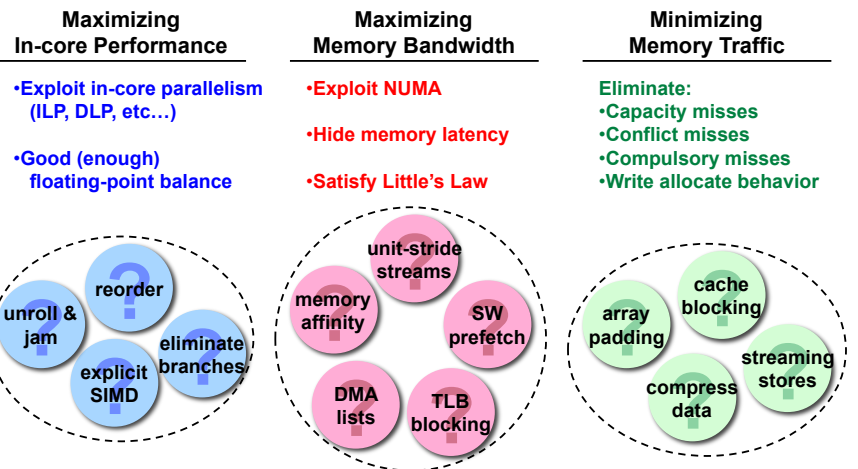


- ❖ Remember, memory traffic includes more than just compulsory misses.
- ❖ As such, actual arithmetic intensity may be substantially lower.
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$$AI = \frac{\text{FLOPs}}{\text{Conflict} + \text{Capacity} + \text{Allocations} + \text{Compulsory}}$$

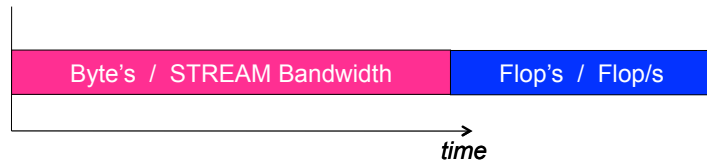


- ❖ Optimizations remove these walls and ceilings which act to constrain performance.

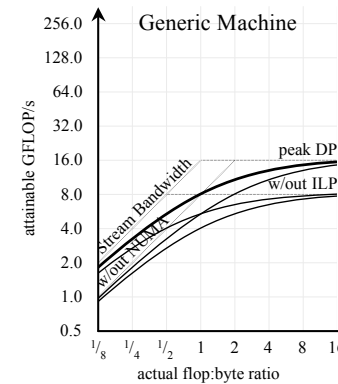




- ❖ Previously, we assumed perfect overlap of communication or computation.
- ❖ What happens if there is a dependency (either inherent or by a lack of optimization) that serializes communication and computation ?



- ❖ Time is the sum of communication time and computation time.
- ❖ **The result is that flop/s grows asymptotically.**

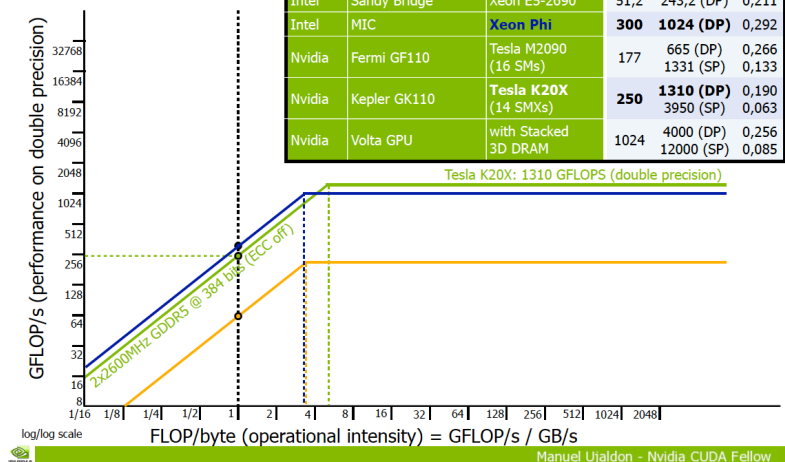


- ❖ Consider a generic machine
- ❖ If we can perfectly decouple and overlap communication with computation, the roofline is sharp/angular.
- ❖ However, without overlap, the roofline is smoothed, **and attainable performance is degraded by up to a factor of 2x.**

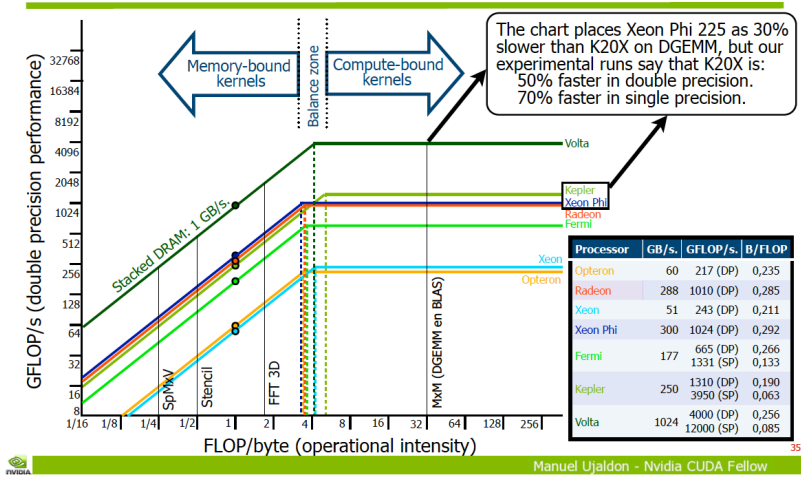
- ❖ Thus far, we assumed a synergy between streaming applications and bandwidth (proxied by the STREAM benchmark)
- ❖ **STREAM is NOT a good proxy for short stanza/random cacheline access patterns as memory latency (instead of just bandwidth) is being exposed.**
- ❖ Thus one might conceive of alternate memory benchmarks to provide a bandwidth upper bound (ceiling)
- ❖ Similarly, if data is primarily local in the LLC cache, one should construct rooflines based on LLC bandwidth and flop:LLC byte ratios.
- ❖ For GPUs/accelerators, PCIe bandwidth can be an impediment. Thus one can construct a roofline model based on PCIe bandwidth and the flop:PCIe byte ratio.

**Platforms to compare**

Vendor	Microarchitecture	Model	GB/s.	GFLOP/s.	Byte/FLOP
AMD	Bulldozer	<b>Opteron 6284</b>	59,7	217,6 (DP)	0,235
AMD	Souther Islands	Radeon HD7970	288	1010 (DP)	0,285
Intel	Sandy Bridge	Xeon E5-2690	51,2	243,2 (DP)	0,211
Intel	MIC	<b>Xeon Phi</b>	300	<b>1024 (DP)</b>	0,292
Nvidia	Fermi GF110	Tesla M2090 (16 SMs)	177	665 (DP) 1331 (SP)	0,266 0,133
Nvidia	Kepler GK110	<b>Tesla K20X</b> (14 SMXs)	250	<b>1310 (DP)</b> 3950 (SP)	0,190 0,063
Nvidia	Volta GPU	with Stacked 3D DRAM	1024	4000 (DP) 12000 (SP)	0,256 0,085



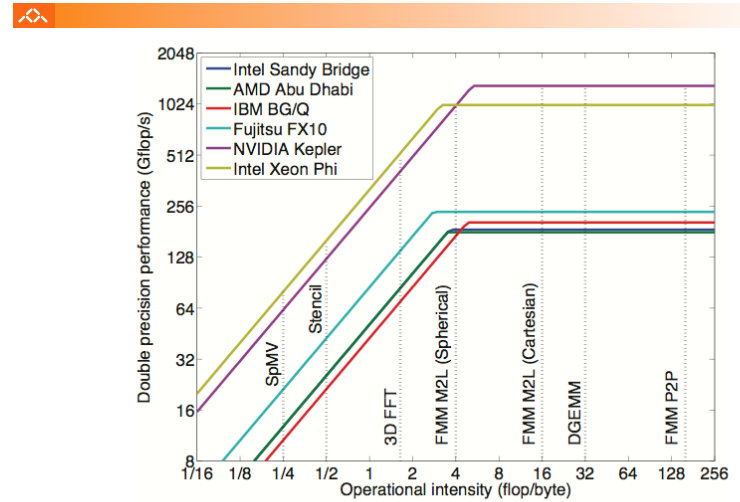
# The Roofline model: Hardware vs. Software



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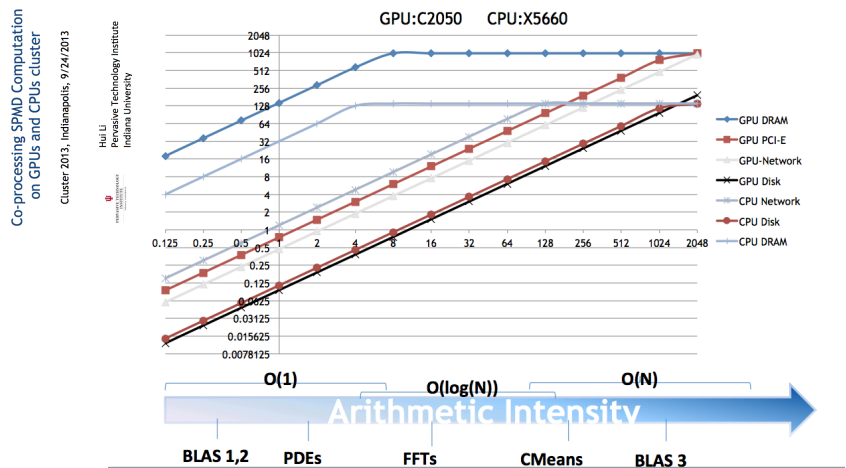
# Some more examples



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# Some more examples



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## Alternate Computations

FUTURE TECHNOLOGIES GROUP

The Roofline Model

Samuel Williams

- ❖ Arising from HPC kernels, its no surprise roofline use DP Flop/s.
- ❖ Of course, it could use
  - SP flop/s,
  - integer ops,
  - bit operations,
  - pairwise comparisons (sorting),
  - graphics operations,
  - etc...