Introduction - Profile Extrusion
Introduction - Design Approaches

% Time

Traditional

Current

Future

Outline

• Extrusion Dies
  • Problem Statement
  • Flow Distribution Optimisation
  • Flow Balance Strategies
  • Optimisation
  • Length vs Thickness Optimisation
  • Conclusion

• Calibrators
  • Problem Statement
  • System Behaviour
  • Optimisation Methodology
  • Case Study
  • Conclusion

• Conclusion

• Ongoing Work
Extrusion Dies – Problem Statement

Unbalanced
Balanced

Extrusion run

Numerical Velocity contours

Extrusion Dies – Flow Distribution Optimisation

Trial Parameters

Pre-Processor

Geometry
Mesh

3D non-isothermal flow field calculation (FVM)

Velocity
Pressure
Temperature

Performance Evaluation

Modification of the controllable geometrical parameters until the optimum is reached
Extrusion Dies – Flow Distribution Optimisation

**Trial Parameters**

**Pre-Processor**

3D non-isothermal flow field calculation (FVM)

3D non-isothermal flow field calculation (FVM)

**Performance Evaluation**

Conservation of mass:

$$\frac{\partial \rho u_j}{\partial x_j} = 0$$

Conservation of linear momentum:

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \tau_{ij} \right)$$

Conservation of energy:

$$\frac{\partial p c_i T}{\partial t} + \frac{\partial p c_i u_i T}{\partial x_j} = \frac{\partial}{\partial x_i} \left( \kappa \frac{\partial T}{\partial x_i} \right) + \tau_{ij} \frac{\partial u_i}{\partial x_j}$$

Constitutive equation (Gen. Newtonian):

$$\tau_{ij} = \eta \left( \dot{\gamma} \right) \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

**Progressive mesh refinements**

<table>
<thead>
<tr>
<th>Cells along Thickness</th>
<th>Number of Cells</th>
<th>Time [h:m:s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>15 496</td>
<td>0:00:36</td>
</tr>
<tr>
<td>4</td>
<td>92 248</td>
<td>0:12:15</td>
</tr>
<tr>
<td>6</td>
<td>272 220</td>
<td>1:12:17</td>
</tr>
<tr>
<td>8</td>
<td>593 928</td>
<td>4:28:36</td>
</tr>
<tr>
<td>10</td>
<td>688 024</td>
<td>6:43:42</td>
</tr>
</tbody>
</table>

PIV / 2.4 GHz
**Conservation of mass:**
\[
\frac{\partial \rho u_j}{\partial x_j} = 0
\]

**Conservation of linear momentum:**
\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_j u_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \tau_{ij} \right)
\]

**Conservation of energy:**
\[
\frac{\partial \rho c T}{\partial t} + \frac{\partial \rho c u_j T}{\partial x_j} = \frac{\partial}{\partial x_i} \left( k \frac{\partial T}{\partial x_i} \right) + \tau_{ij} \frac{\partial u_i}{\partial x_j}
\]

**Constitutive equation (viscoelastic):**
\[
\tau_i + \lambda \left( \frac{\partial \tau_i}{\partial t} + \frac{\partial}{\partial x_j} (u_j \tau_i) \right) = \eta_i \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \lambda \left( \tau_{ij} \frac{\partial u_i}{\partial x_j} + \tau_{ij} \frac{\partial u_j}{\partial x_i} \right)
\]
Extrusion Dies – Flow Distribution Optimisation

Trial Parameters
Pre-Processor

Geometry
Mesh

3D non-isothermal flow field calculation (FVM)

Velocity
Pressure
Temperature

Performance Evaluation

Modification of the controllable geometrical parameters until the optimum is reached

SIMPLEX Method (SM)

Experimental Method (EM)

Extrusion Dies – Flow Balance Strategies

Required Profile
Optimised Variable
Die Flow Channel
Haul-off Speed
Final Profile

Length
Thickness

\[ V_3 \neq \frac{V_3}{V_1} = \frac{V_3}{V_2} \]
Extrusion Dies – Optimisation

Initial flow channel dimensions

<table>
<thead>
<tr>
<th>ES</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_i$ [mm]</td>
<td>2.0</td>
<td>2.5</td>
<td>2.5</td>
<td>3.0</td>
<td>2.0</td>
<td>4.0</td>
</tr>
<tr>
<td>$L_i$ [mm]</td>
<td>30.0</td>
<td>37.5</td>
<td>37.5</td>
<td>45.0</td>
<td>30.0</td>
<td>60.0</td>
</tr>
<tr>
<td>$L_i/t_i$</td>
<td>15.0</td>
<td>15.0</td>
<td>15.0</td>
<td>15.0</td>
<td>15.0</td>
<td>15.0</td>
</tr>
</tbody>
</table>
Constitutive equation

\[ \eta(\dot{\gamma}, T) = F(\dot{\gamma} \times H(T))H(T) \]
\[ F(\dot{\gamma}) = \eta_0 + \frac{\eta_0 - \eta_\infty}{(1 + (\lambda\dot{\gamma})^n)} \]
\[ H(T) = \exp\left[\frac{1}{\tau} \right] \]

Mesh

Operating and thermal boundary conditions

<table>
<thead>
<tr>
<th>Condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow rate</td>
<td>20 kg/h</td>
</tr>
<tr>
<td>Melt inlet temperature</td>
<td>230 °C</td>
</tr>
<tr>
<td>Outer die walls temperature</td>
<td>230 °C</td>
</tr>
<tr>
<td>Inner (mandrel) die walls Adiabatic</td>
<td></td>
</tr>
</tbody>
</table>

Extrusion Dies – Optimisation

DieINI – Initial trial

Optimizations performed

DieL – Length optimisation
DieT – Thickness optimisation
DieLS – Length optimisation + Flow separators
Extrusion Dies – Optimisation

DieL

DieT

DieLS

Extrusion Dies – Optimisation

DieIni

DieL

DieT

DieLS

Velocity [m/s]

0.000 0.004 0.008 0.012 0.016 0.020 0.024 0.027 0.031
Extrusion Dies – Optimisation

DieIni

DieL

DieT

Extrusion Dies – Optimisation

Results

<table>
<thead>
<tr>
<th>Length/Thickness</th>
<th>Optimized die</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DieL (length)</td>
<td>DieT (thickness)</td>
<td>DieLS (length)</td>
</tr>
<tr>
<td>ES1</td>
<td>3.75</td>
<td>12.40</td>
<td>10.75</td>
</tr>
<tr>
<td>ES2/3</td>
<td>4.60</td>
<td>14.20</td>
<td>13.80</td>
</tr>
<tr>
<td>ES4</td>
<td>5.83</td>
<td>15.57</td>
<td>12.67</td>
</tr>
<tr>
<td>ES5</td>
<td>3.50</td>
<td>12.40</td>
<td>12.25</td>
</tr>
<tr>
<td>ES6</td>
<td>15.00</td>
<td>18.81</td>
<td>15.00</td>
</tr>
</tbody>
</table>
The factors considered can be divided in two different groups:

i) **processing conditions**: $V, T_w$

ii) **melt rheological properties**: $n$

The experiments (simulations) performed were defined by a statistics Taguchi technique, considering three levels for each factor.
### Extrusion Dies - *Length vs Thickness Optimisation*

#### Ratio $\frac{V_{\text{max}}}{\overline{V}}$

<table>
<thead>
<tr>
<th>Extrusion Die</th>
<th>ES1</th>
<th>ES2</th>
<th>ES3</th>
<th>ES4</th>
<th>ES5</th>
<th>ES6</th>
</tr>
</thead>
<tbody>
<tr>
<td>DieINI</td>
<td>6.20</td>
<td>3.72</td>
<td>3.39</td>
<td>2.18</td>
<td>7.46</td>
<td>1.00</td>
</tr>
<tr>
<td>DieL</td>
<td>1.08</td>
<td>1.15</td>
<td>1.03</td>
<td>1.12</td>
<td>1.15</td>
<td>1.00</td>
</tr>
<tr>
<td>DieT</td>
<td>1.68</td>
<td>1.38</td>
<td>1.33</td>
<td>1.24</td>
<td>1.56</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Extrusion Dies - Conclusion

- **Length control** is difficult to apply in geometries with different flow restrictions and leads to dies with higher sensitivity to processing conditions than thickness control;
- **Flow separators** had a positive effect in the flow distribution but affect negatively in the die sensitivity;
- **Thickness optimised dies** produce extrudates that have higher propensity to distort.

Calibrators – Problem Statement

\[ \bar{T} < T_s \downarrow \sigma_T \]
Calibrators - Pre-processor

- Profile cross section
- No. of calibrators
- For each calibrator:
  - Dimensions
  - Axial location
  - Length
  - No. of cooling channels
  - Dimension of the cooling
  - Layout of the cooling

Calibrators – Numerical boundary conditions

- Temperature Imposed
- Adiabatic
- Contact resistance
- Extrusion Direction
- Free convection and radiation or adiabatic
**Calibrators - Equations to solve**

**Polymer**

\[
\frac{\partial}{\partial x} \left( k_p \frac{\partial T_p}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_p \frac{\partial T_p}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_p \frac{\partial T_p}{\partial z} \right) - \rho_p c_p \frac{\partial}{\partial z} (\omega T_p) = 0
\]

**Calibrator**

\[
\frac{\partial}{\partial x} \left( k_c \frac{\partial T_c}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_c \frac{\partial T_c}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_c \frac{\partial T_c}{\partial z} \right) = 0
\]

**Polymer-calibrator interface**

*Contact Resistance*

\[
k_c \left( \frac{\partial T_c}{\partial n} \right)_{\text{interface}} = -k_p \left( \frac{\partial T_p}{\partial n} \right)_{\text{interface}} = h \left( T_p - T_c \right)_{\text{interface}}
\]

**Calibrators - Typical result**

3D Temperature field calculation (FVM)
Influence of boundary conditions, process and geometrical parameters on the system performance (in terms of average temperature and temperature uniformity)

**Conclusion:**

In general

![Diagram showing the relationship between average temperature (\(\overline{T}\)) and temperature uniformity (\(\sigma_T\)).](image)

Exceptions

Influence of the cooling units and annealing zones lengths and cooling fluid temperature on the system performance.

Influence of Length Distribution LCi and Dij
Calibrators - System Behaviour

![Diagram of calibrator system]

| \( \text{LCi} \) (600 mm), \( \Sigma \text{D} \) (240 mm) (system length = 850 mm) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| \( \text{LCi} \) | \( \text{LC1} \) | \( \text{LC2} \) | \( \text{LC3} \) | \( \text{Dij} \) | \( \text{D12} \) | \( \text{D23} \) |
| [mm] | [mm] | [mm] | [mm] | [mm] | [mm] | [mm] |
| 600 | - | - | - | - | - | - |
| 300 | 300 | - | - | 240 | - | - |
| 200 | 200 | 200 | - | 120 | 120 | - |
| 200 | 200 | 200 | - | 60 | 180 | - |
| 300 | 200 | 100 | - | 120 | 120 | - |
| 100 | 200 | 300 | - | 120 | 120 | - |
| 300 | 200 | 100 | - | 120 | 120 | - |
| 100 | 200 | 300 | - | 180 | 60 | - |
| 100 | 200 | 300 | - | 60 | 180 | - |

Calibrators - System Behaviour

![Diagram of calibrator system]

| \( \text{LCi} \) (600 mm), \( \Sigma \text{D} \) (240 mm) (system length = 850 mm) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| \( \text{LCi} \) | \( \text{LC1} \) | \( \text{LC2} \) | \( \text{LC3} \) | \( \text{Dij} \) | \( \text{D12} \) | \( \text{D23} \) | \( \bar{T} \) | \( \Delta \tau \) | \( \sigma \tau \) |
| [mm] | [mm] | [mm] | [mm] | [mm] | [mm] | [mm] | [ºC] | [%] | [ºC] | [%] |
| 600 | - | - | - | - | - | - | 84.9 | 0.0% | 16.6 | 0.0% |
| 300 | 300 | - | - | 240 | - | - | 80.3 | -5.5% | 15.2 | -8.6% |
| 200 | 200 | 200 | - | 120 | 120 | - | 79.2 | -6.7% | 14.5 | -12.6% |
| 200 | 200 | 200 | - | 60 | 180 | - | 79.5 | -6.4% | 14.5 | -13.1% |
| 200 | 200 | 200 | - | 180 | 60 | - | 79.4 | -6.5% | 14.8 | -10.8% |
| 300 | 200 | 100 | - | 120 | 120 | - | 79.5 | -6.4% | 13.0 | -22.1% |
| 100 | 200 | 300 | - | 120 | 120 | - | 79.4 | -6.5% | 15.1 | -9.3% |
| 300 | 200 | 100 | - | 180 | 60 | - | 79.6 | -6.3% | 13.8 | -17.3% |
| 300 | 200 | 100 | - | 60 | 180 | - | 79.9 | -5.9% | 12.6 | -24.3% |
| 100 | 200 | 300 | - | 180 | 60 | - | 79.7 | -6.1% | 15.2 | -8.4% |
| 100 | 200 | 300 | - | 60 | 180 | - | 79.5 | -6.3% | 15.1 | -9.4% |

\( \Sigma \text{LCi} \) (600 mm), \( \Sigma \text{D} \) (240 mm) (system length = 850 mm)
Calibrators - System Behaviour

Influence of cooling fluid temperature
TCi
### Calibrators - System Behaviour

#### Temperature Calibration

<table>
<thead>
<tr>
<th>TCi</th>
<th>TC1</th>
<th>TC2</th>
<th>TC3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[°C]</td>
<td>[°C]</td>
<td>[°C]</td>
</tr>
</tbody>
</table>

- **LCi + Dij**
  - 10ºC ≤ T Ci ≤ 26ºC

<table>
<thead>
<tr>
<th>TCi</th>
<th>TC1</th>
<th>TC2</th>
<th>TC3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[°C]</td>
<td>[°C]</td>
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</table>

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### Calibrators - System Behaviour

#### Temperature Calibration

<table>
<thead>
<tr>
<th>TCi</th>
<th>TC1</th>
<th>TC2</th>
<th>TC3</th>
<th>T̅</th>
<th>σ_T</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[°C]</td>
<td>[°C]</td>
<td>[°C]</td>
<td>[°C]</td>
<td>[%]</td>
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- **LCi + Dij**
  - 10ºC ≤ T Ci ≤ 26ºC

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<th>T̅</th>
<th>σ_T</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[°C]</td>
<td>[°C]</td>
<td>[°C]</td>
<td>[°C]</td>
<td>[%]</td>
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- **LCi + Dij**
  - 10ºC ≤ T Ci ≤ 26ºC

### Calibrators - System Behaviour

#### Temperature Calibration

<table>
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<th>TC3</th>
<th>T̅</th>
<th>σ_T</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[°C]</td>
<td>[°C]</td>
<td>[°C]</td>
<td>[°C]</td>
<td>[%]</td>
</tr>
</tbody>
</table>

- **LCi + Dij**
  - 10ºC ≤ T Ci ≤ 26ºC

<table>
<thead>
<tr>
<th>TCi</th>
<th>TC1</th>
<th>TC2</th>
<th>TC3</th>
<th>T̅</th>
<th>σ_T</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[°C]</td>
<td>[°C]</td>
<td>[°C]</td>
<td>[°C]</td>
<td>[%]</td>
</tr>
</tbody>
</table>

- **LCi + Dij**
  - 10ºC ≤ T Ci ≤ 26ºC
Calibrators - System Behaviour

Calibrators - Optimisation Methodology

Input Data

Pre-Processor

Geometry

Mesh

3D Temperature field calculation (FVM)

Temperature

Performance Evaluation

Optimisation: automatic generation of solutions (modification of the controllable geometrical parameters) until the optimum is reached
Calibrators - Optimisation Methodology

**Input Data**

**Pre Processor**

**Geometry**

**Mesh**

**3D Temperature field calculation (FVM)**

**Temperature**

**Performance Evaluation**

\[ \frac{\sum_i (T_i - \bar{T}) A_i}{A_T} \]

**Optimisation method**

- **Temperature uniformity**
  - \( \sigma_T = \frac{1}{A_T} \sum_{i=1}^{n} (T_i - \bar{T}) A_i \)
- **Cooling efficiency**
  - \( T = \frac{\sum_{i=1}^{n} T_i A_i}{A_T} \)

**Objective Function**

\[
F_{obj} = K \left( \bar{T} - T_s \right) + \sigma_T
\]

where:

- \( \bar{T} \leq T_s \Rightarrow K = 0 \)
- \( \bar{T} > T_s \Rightarrow K = 1000 \)

**Optimisation algorithm**

Non-linear SIMPLEX method
Calibrators - Case Study

Restrictions:

- Number of calibration/cooling units <= 3
- Total calibration length (∑LCi) <= 600 mm
- Total system length (∑LCi + ∑Dij + 10) <= 850 mm
- Cooling Fluid Temperature TCi ∈ [10ºC,26ºC]

Calibrators - Case Study

General conditions for the simulations

**Processing conditions**
- v_p = 2 m/min
- T_m = 180 ºC
- T_f = 18 ºC
- T_s = 80 ºC

**Materials Properties**
- K_p = 0.18 W/mK
- K_c = 14 W/mK
- ρ_p = 1400 kg/m³
- c_p = 1000 J/kgK

**Boundary conditions**
- Annealing zones: free convection and radiation
- Polymer-calibrator interface: contact resistance
  (h_i = 425 W/m²K)
Geometry

Mesh

Calibrators - Case Study

Local minimum?
Calibrators - Case Study

TC1 = 10°C
TC2 = 26°C

Calibrators - Case Study

<table>
<thead>
<tr>
<th>Geometry</th>
<th>$\bar{T}$ [°C]</th>
<th>$\sigma_T$ [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Calibrator (TC=18°C)</td>
<td>84.9</td>
<td>16.6</td>
</tr>
<tr>
<td>Optimum Solution</td>
<td>78.5</td>
<td>7.9</td>
</tr>
</tbody>
</table>

- 52.4%
• Cooling systems with
  ascending cooling units lengths
  descending annealing zone lengths
  ascending cooling fluid temperatures
seem to have the best performance.

• The developed optimisation methodologies both for extrusion dies and calibrators were able to improve automatically the system performance;
• The optimisation methodologies are under development;
• The employment of numerical analysis allows a deeper insight of the process.
Ongoing work

- Implementation of the wall slip and free-surface boundary conditions (L.L. Ferrás, PhD project);
- Development of unstructured numerical modelling code (N.D. Gonçalves, PhD project);
- Prediction of thermal induced stresses (A.M. Ribau, FCT Research project);
- Development of multiscale modelling approaches (S.T. Mould, PhD Project);
- Development of SPH numerical modelling code (D.F. Cordeiro, PhD/Cooperation Project);

Ongoing work

- Numerical code parallelization on GPU (S.P. Pereira, FCT Research Project);
Ongoing work

• Numerical code parallelization on GPU (S.P. Pereira, FCT Research Project);

A colouring scheme was used to avoid race conditions.

Poiseuille Flow – Speed Up
Ongoing work

- Numerical code parallelization on GPU (S.P. Pereira, FCT Research Project);

Lid Driven Cavity Flow – Speed Up