

# Computer Aided Design of Thermoplastic Profile Extrusion Forming Tools

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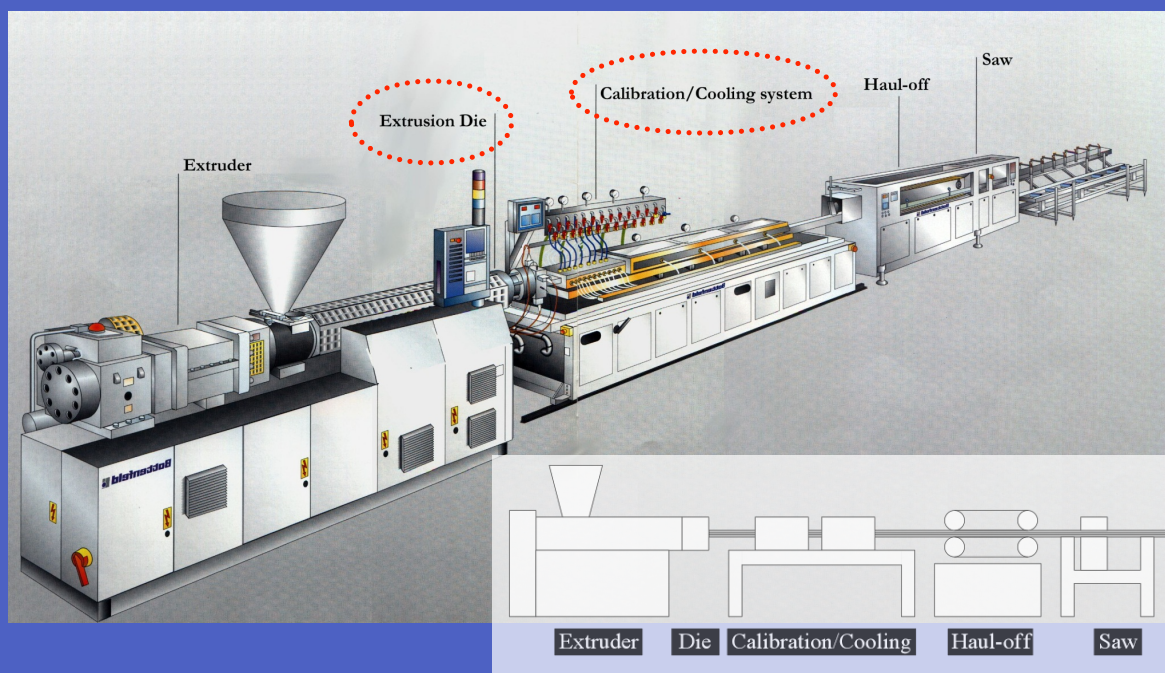


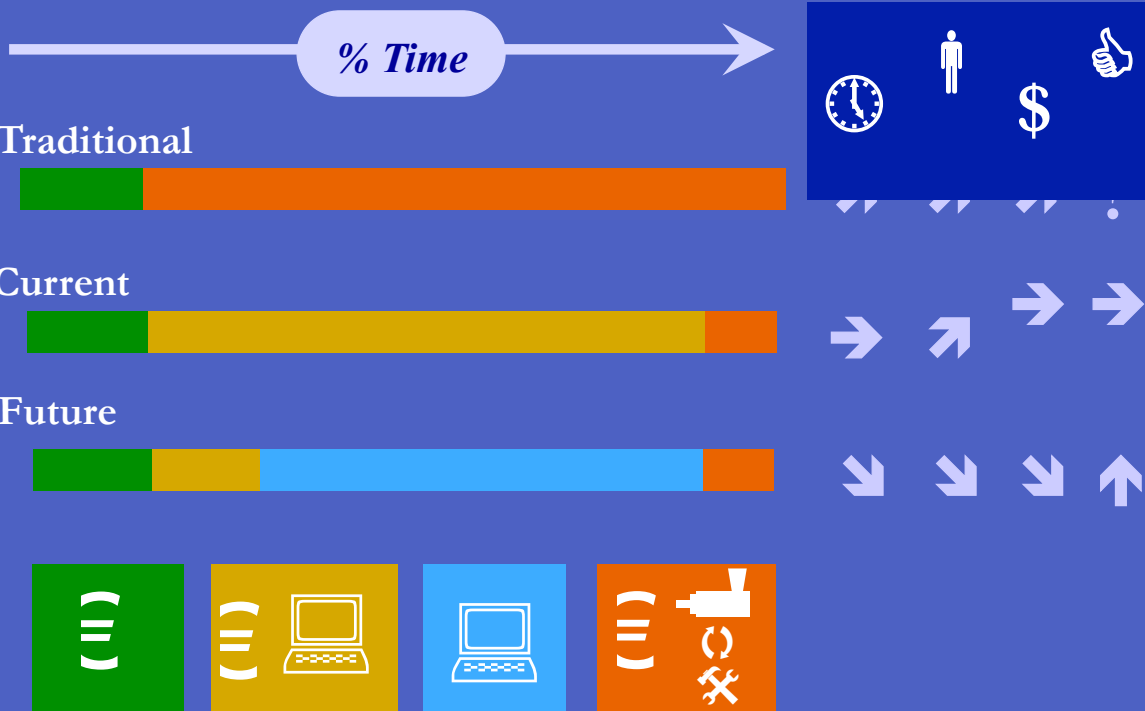
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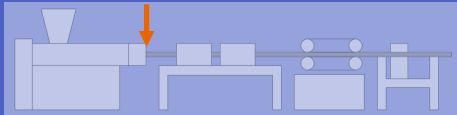
## Introduction - Profile Extrusion





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  - Flow Balance Strategies
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  - Conclusion
- **Calibrators**
  - Problem Statement
  - System Behaviour
  - Optimisation Methodology
  - Case Study
  - Conclusion
- **Conclusion**
- **Ongoing Work**

# Extrusion Dies – Problem Statement



Extrusion run

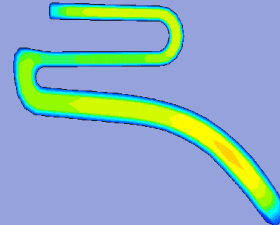
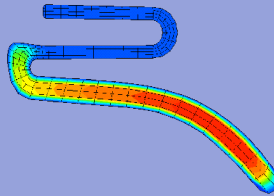
Unbalanced



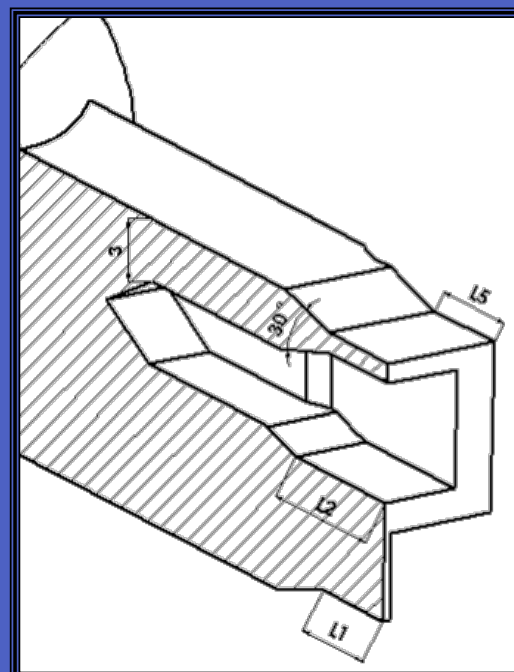
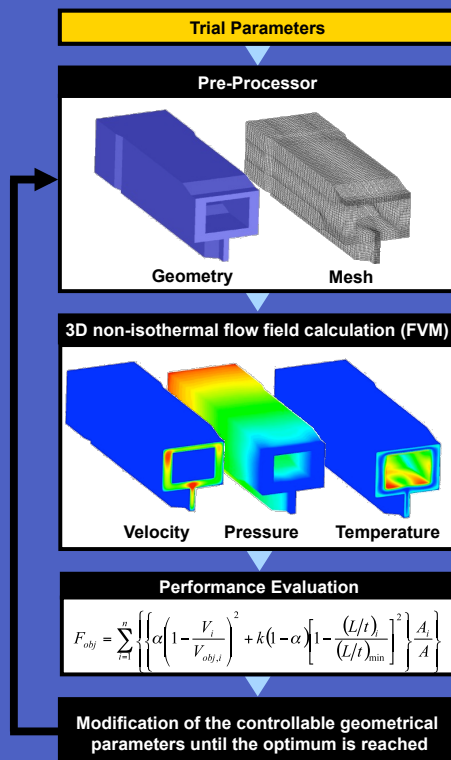
Balanced

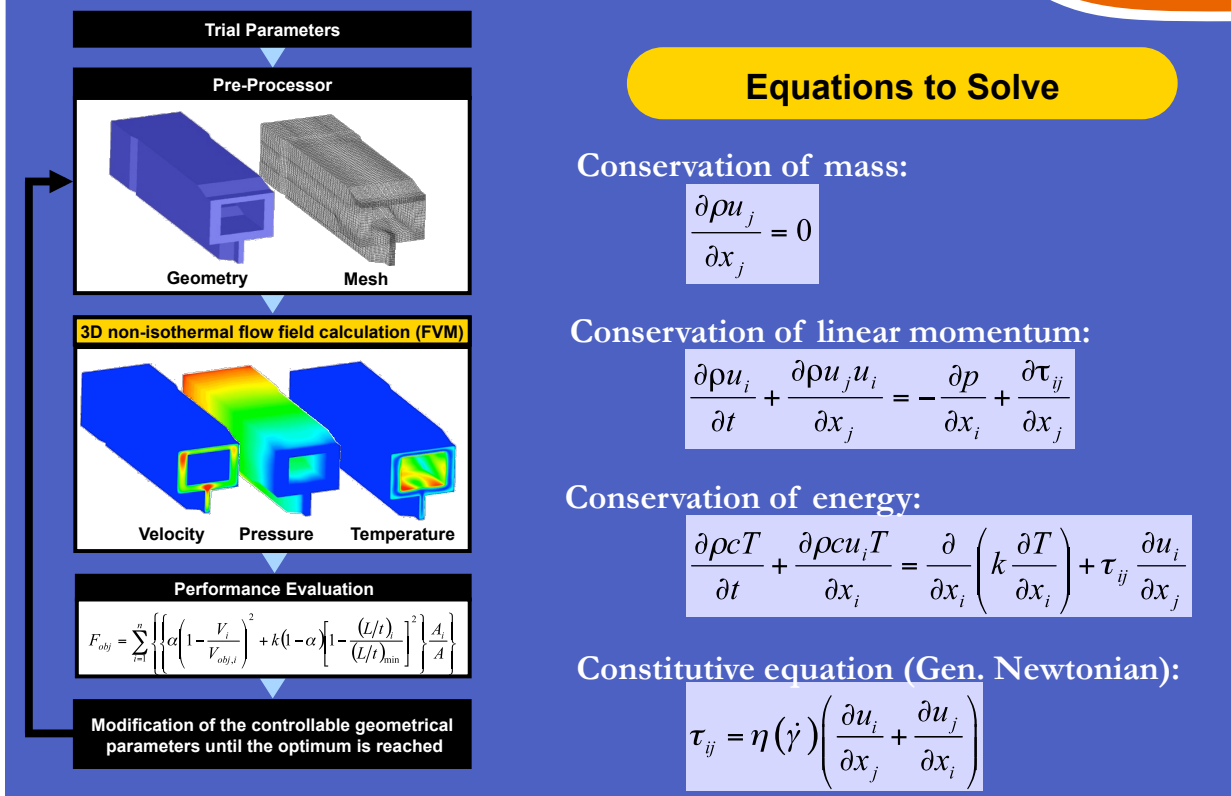
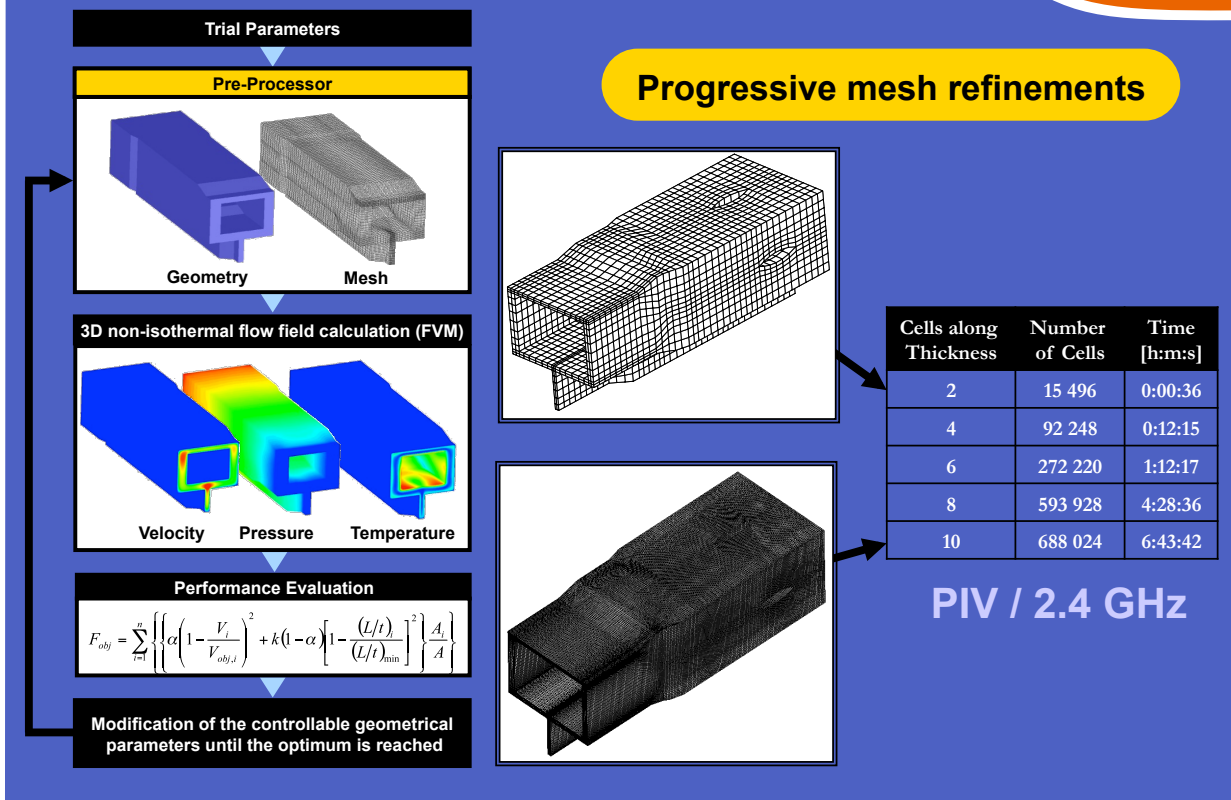


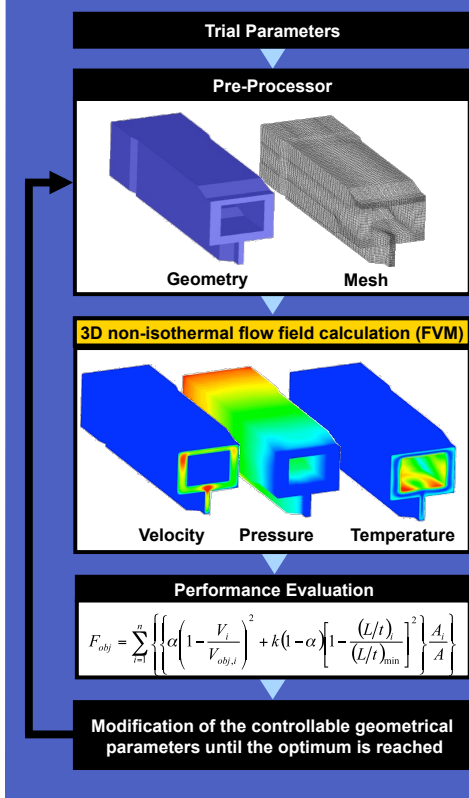
Numerical Velocity contours



# Extrusion Dies – Flow Distribution Optimisation







## Equations to Solve

Conservation of mass:

$$\frac{\partial \rho u_j}{\partial x_j} = 0$$

Conservation of linear momentum:

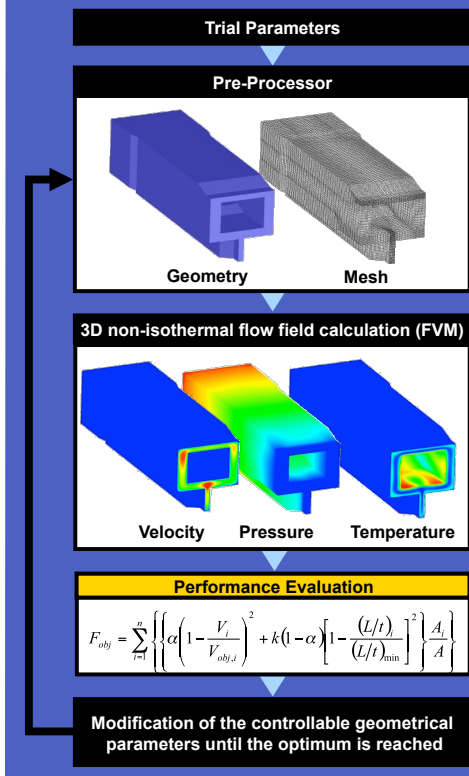
$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_j u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}$$

Conservation of energy:

$$\frac{\partial \rho c T}{\partial t} + \frac{\partial \rho c u_i T}{\partial x_i} = \frac{\partial}{\partial x_i} \left( k \frac{\partial T}{\partial x_i} \right) + \tau_{ij} \frac{\partial u_i}{\partial x_j}$$

Constitutive equation (viscoelastic):

$$\tau_{ij} + \lambda \left( \frac{\partial \tau_{ij}}{\partial t} + \frac{\partial (u_k \tau_{ij})}{\partial x_k} \right) = \eta_p \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \lambda \left( \tau_{jk} \frac{\partial u_i}{\partial x_k} + \tau_{ik} \frac{\partial u_j}{\partial x_k} \right)$$



## Objective Function

$$F_{obj} = \sum_{i=1}^n \left\{ \underbrace{\alpha \left( 1 - \frac{V_i}{V_{obj,i}} \right)^2}_{\text{Flow Balance}} + k \underbrace{(1 - \alpha) \left[ 1 - \frac{(L/t)_i}{(L/t)_{min}} \right]^2}_{\text{Admissible L/t value}} \right\} \frac{A_i}{A}$$

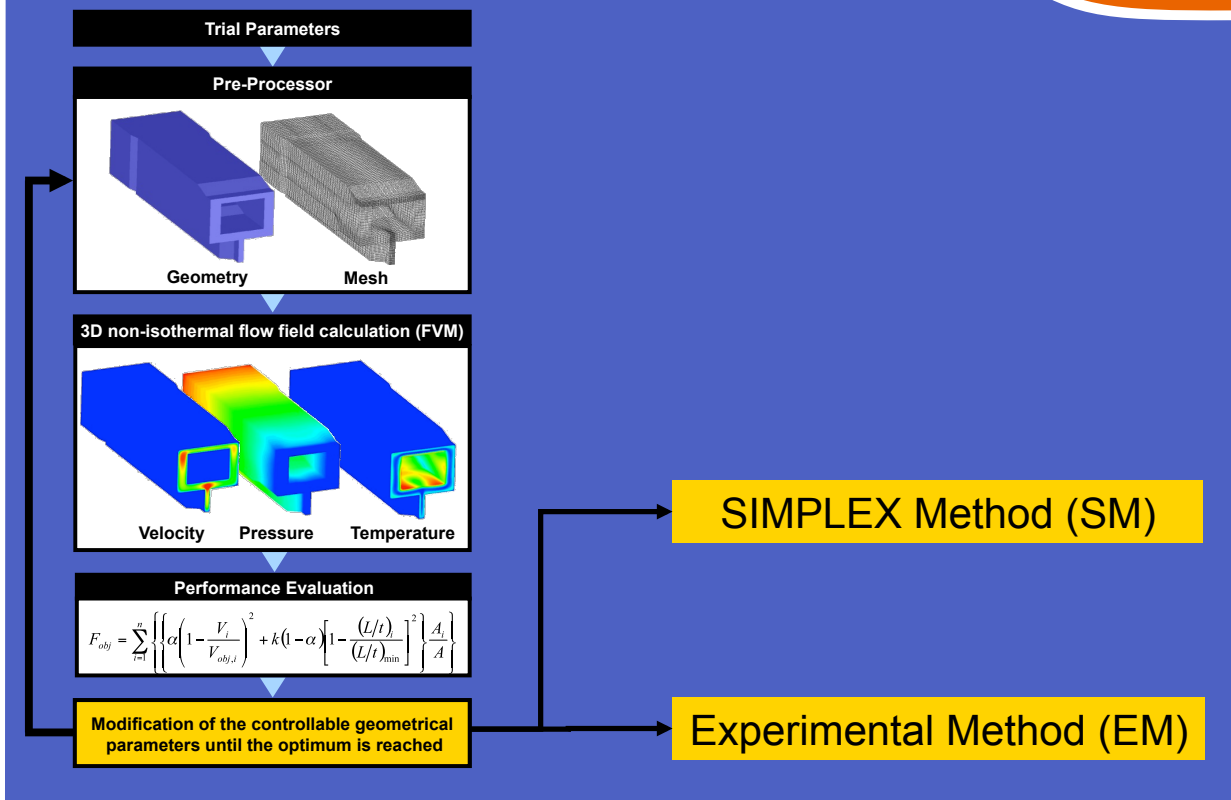
Area Weighting

Flow Balance

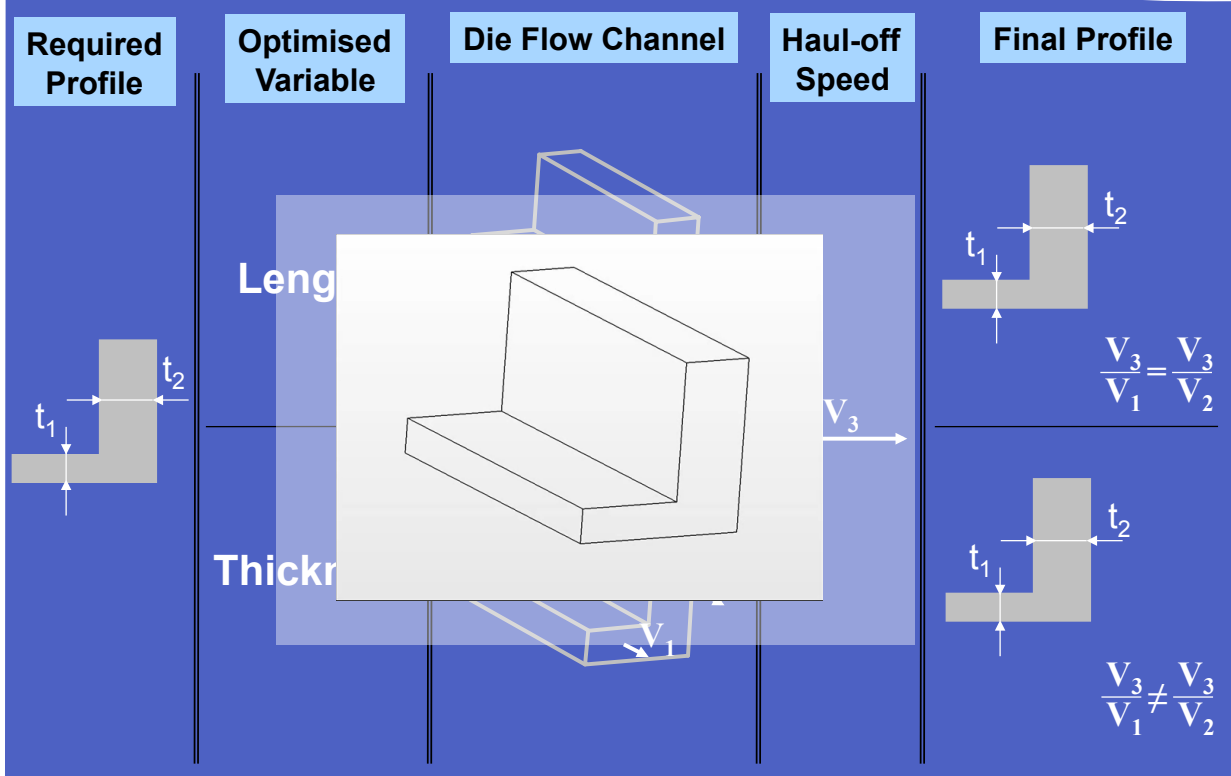
Admissible L/t value

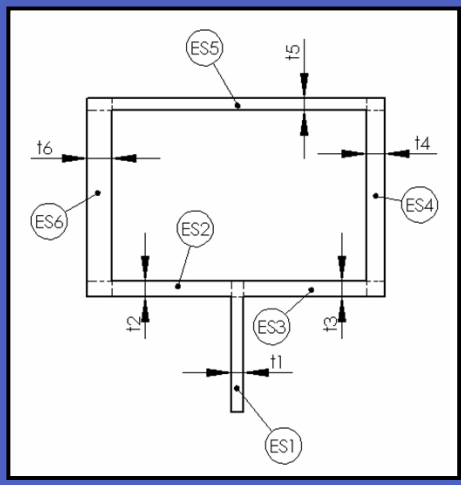
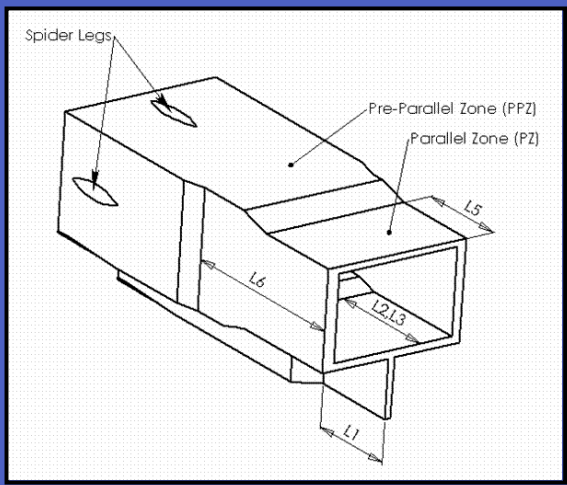
$$F_{obj} = \sum_{i=1}^n \left\{ \alpha \left( 1 - \frac{V_i}{V_{obj,i}} \right)^2 + k (1 - \alpha) \left[ 1 - \frac{(L/t)_i}{(L/t)_{min}} \right]^2 \right\} \frac{A_i}{A}$$

# Extrusion Dies – Flow Distribution Optimisation



# Extrusion Dies – Flow Balance Strategies





Initial flow channel dimensions

ES	1	2	3	4	5	6
$t_i$ [mm]	2.0	2.5	2.5	3.0	2.0	4.0
$L_i$ [mm]	30.0	37.5	37.5	45.0	30.0	60.0
$L_i/t_i$	15.0	15.0	15.0	15.0	15.0	15.0



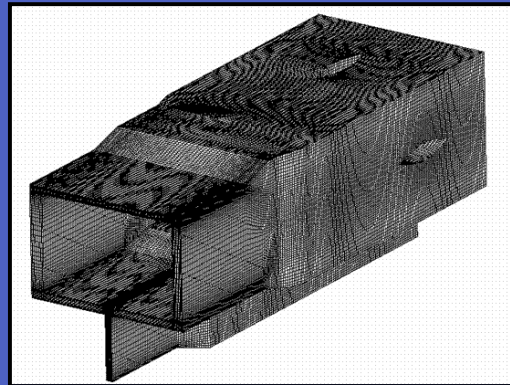


## Constitutive equation

$$\eta(\dot{\gamma}, T) = F(\dot{\gamma} \times H(T))H(T)$$

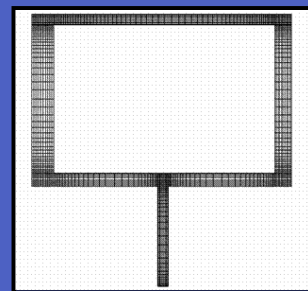
$$F(\dot{\gamma}) = \eta_{\infty} + \frac{\eta_0 - \eta_{\infty}}{(1 + (\lambda\dot{\gamma})^2)^{\frac{1-n}{2}}} \quad H(T) = \exp\left[\alpha\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

## Mesh



## Operating and thermal boundary conditions

Flow rate	20 kg/h
Melt inlet temperature	230 °C
Outer die walls temperature	230 °C
Inner (mandrel) die walls	Adiabatic



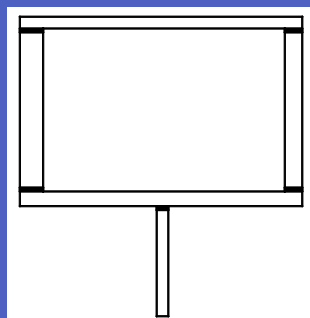
## DieINI – Initial trial

### Optimizations performed

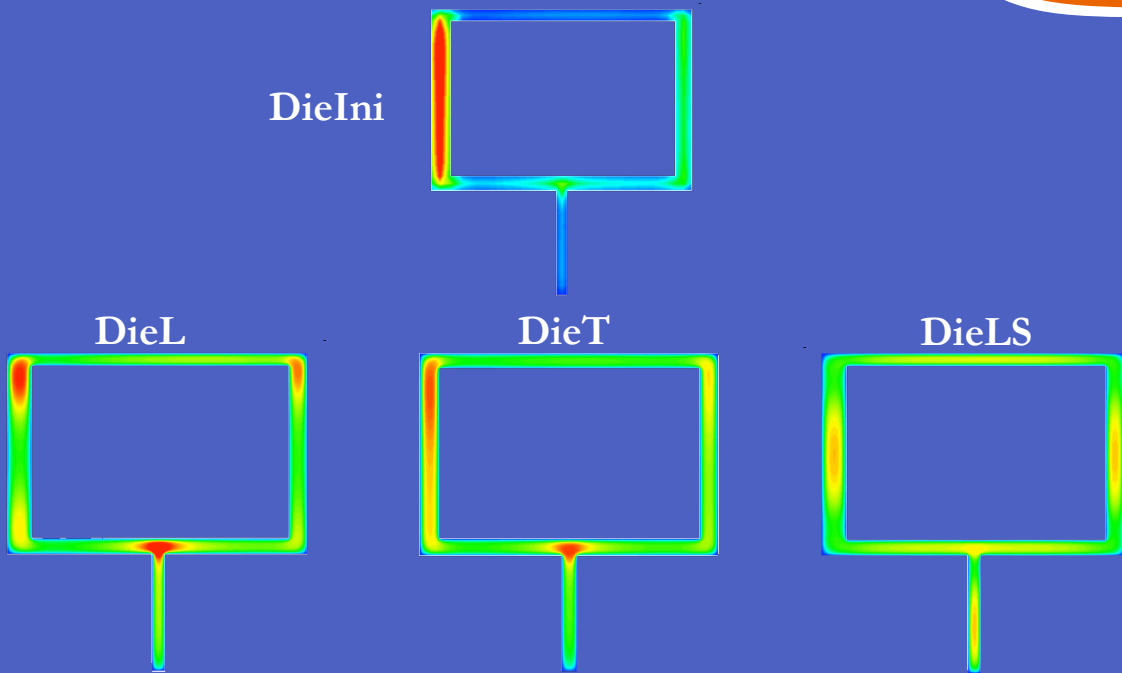
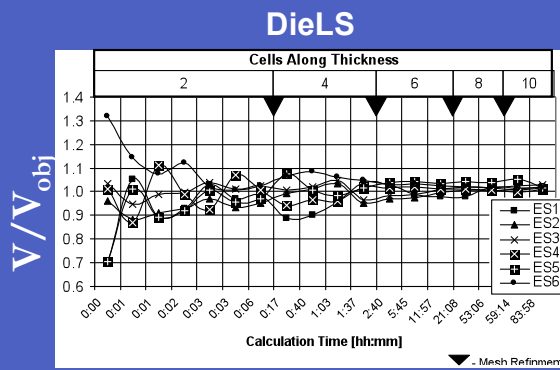
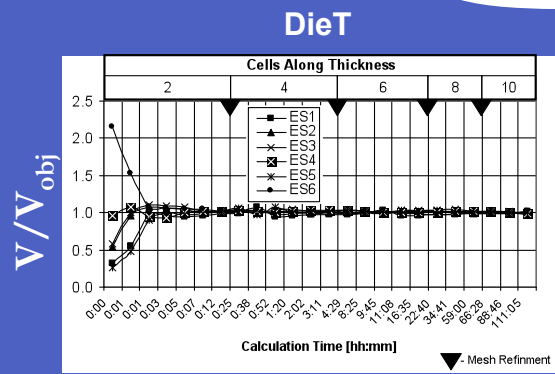
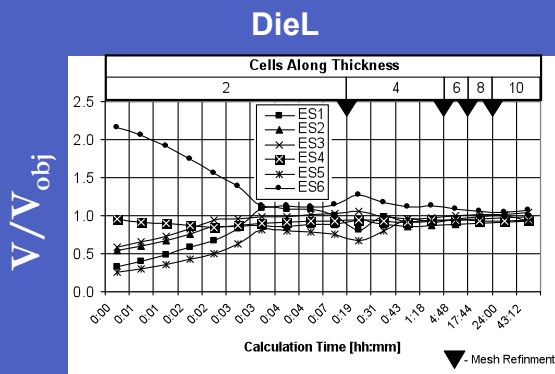
**DieL** – Length optimisation

**DieT** – Thickness optimisation

**DieLS** – Length optimisation + Flow separators

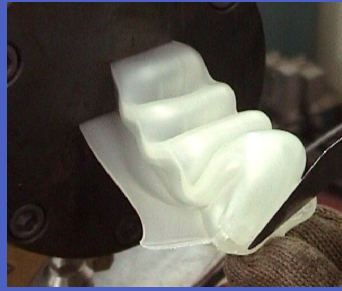








DieIni



DieL



DieT



**Results**

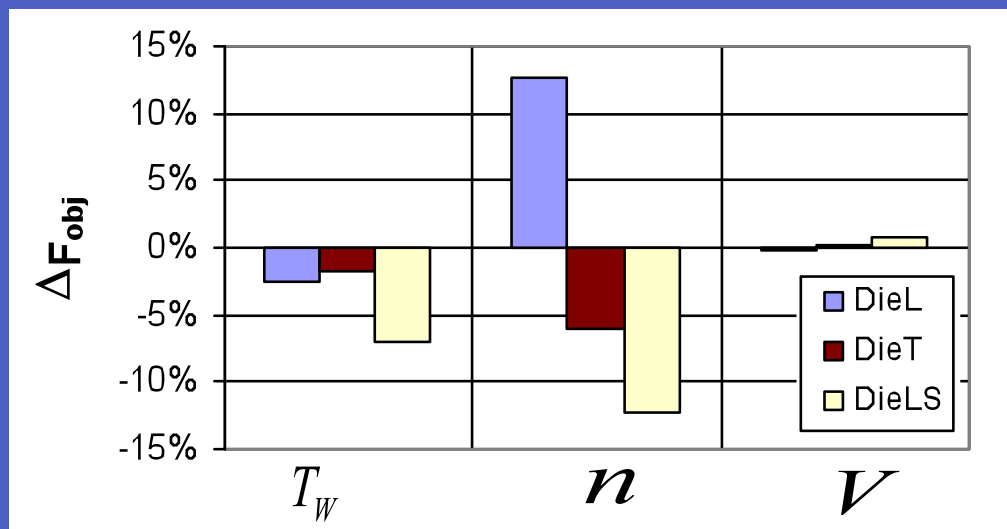
		Optimized die		
		DieL (length)	DieT (thickness)	DieLS (length)
Length/ Thickness	ES1	3.75	12.40	10.75
	ES2/3	4.60	14.20	13.80
	ES4	5.83	15.57	12.67
	ES5	3.50	12.40	12.25
	ES6	15.00	18.81	15.00



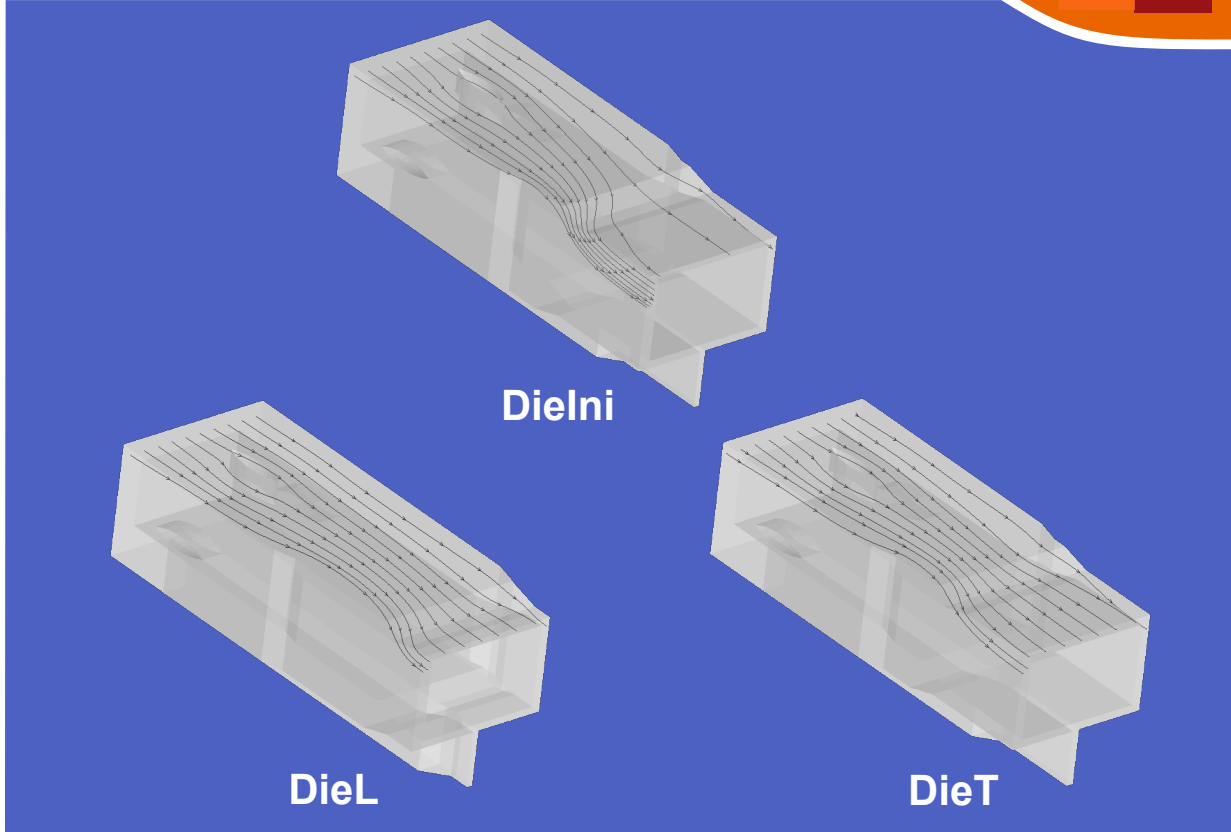
The factors considered can be divided in two different groups:

- i) **processing conditions**:  $V$ ,  $T_w$
- ii) **melt rheological properties**:  $n$

The experiments (simulations) performed were defined by a statistics Taguchi technique, considering three levels for each factor



# Extrusion Dies - Length vs Thickness Optimisation

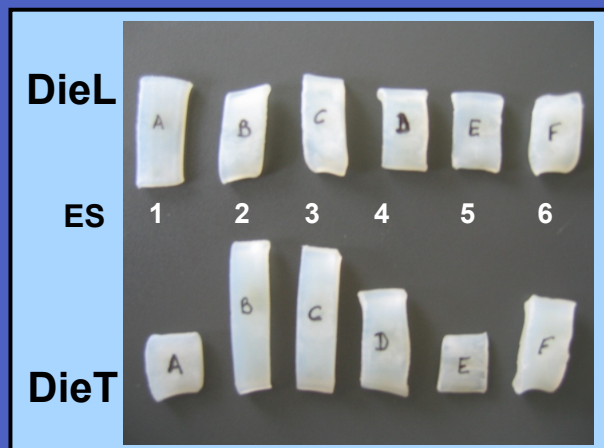
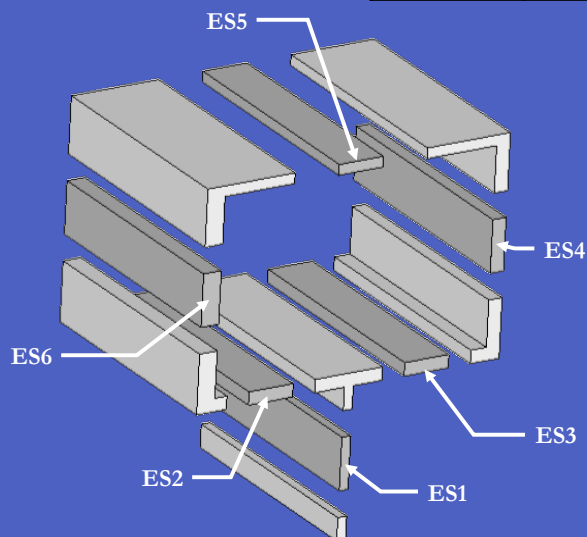


# Extrusion Dies - Length vs Thickness Optimisation



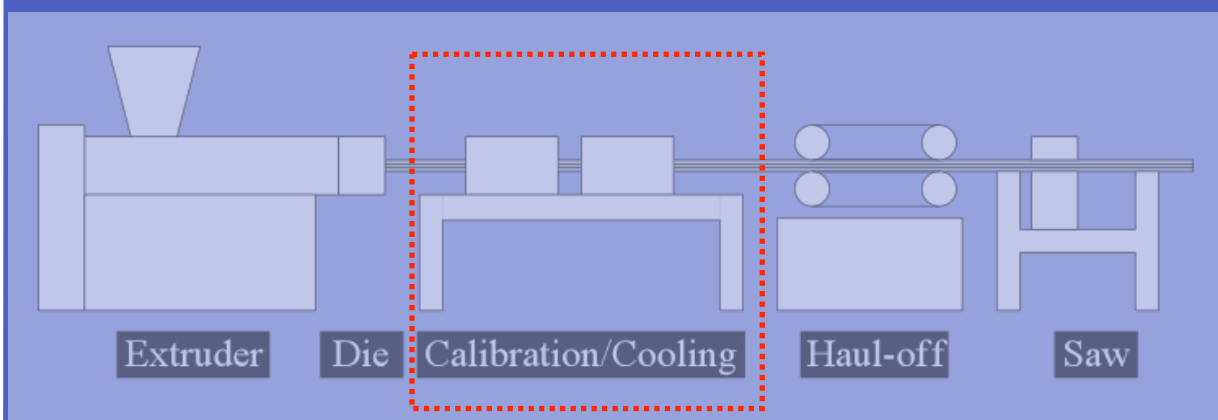
Ratio  $\bar{V}_{max} / \bar{V}$

Extrusion Die	ES1	ES2	ES3	ES4	ES5	ES6
DieINI	6.20	3.72	3.39	2.18	7.46	1.00
DieL	1.08	1.15	1.03	1.12	1.15	1.00
DieT	1.68	1.38	1.33	1.24	1.56	1.00

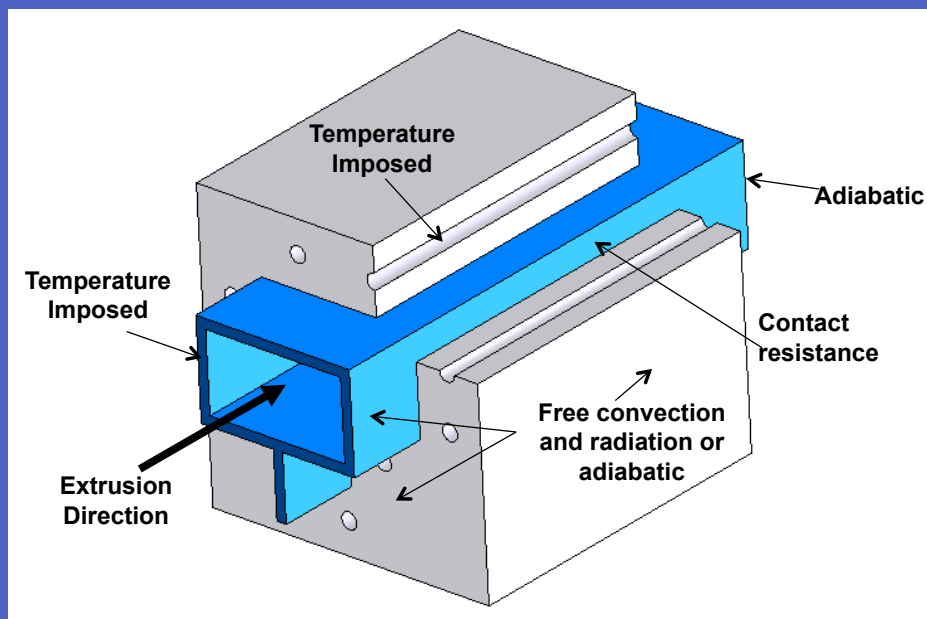
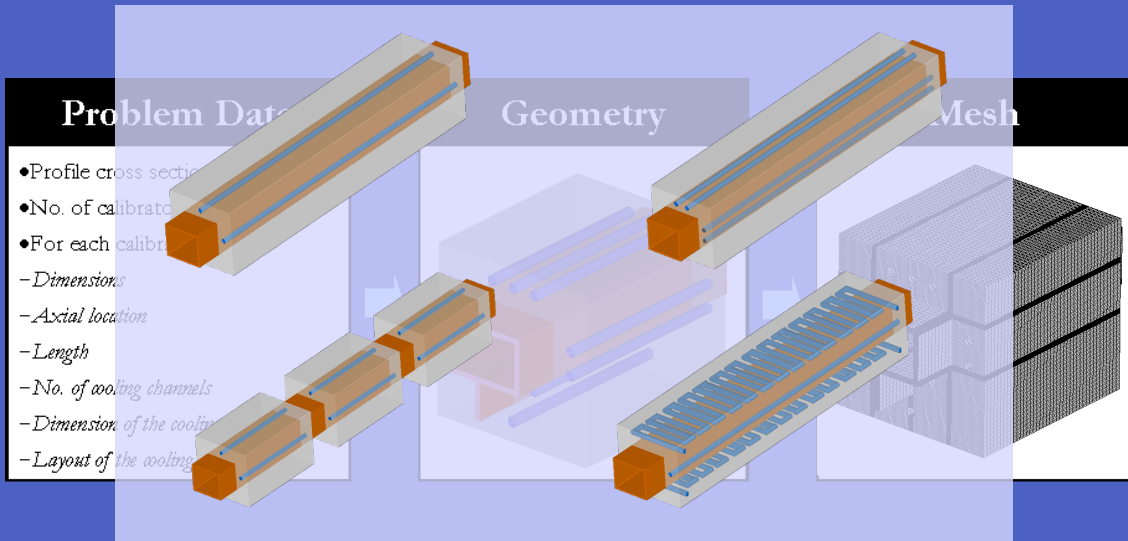




- *Length control* is difficult to apply in geometries with different flow restrictions and leads to dies with higher sensitivity to processing conditions than thickness control;
- *Flow separators* had a positive effect in the flow distribution but affect negatively in the die sensitivity;
- *Thickness optimised dies* produce extrudates that have higher propensity to distort.



$$\bar{T} < T_s \quad \downarrow \quad \sigma_T$$





## Polymer

$$\frac{\partial}{\partial x} \left( k_p \frac{\partial T_p}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_p \frac{\partial T_p}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_p \frac{\partial T_p}{\partial z} \right) - \rho_p c_p \frac{\partial}{\partial z} (w T_p) = 0$$

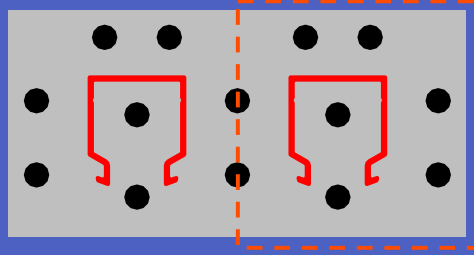
## Calibrator

$$\frac{\partial}{\partial x} \left( k_c \frac{\partial T_c}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_c \frac{\partial T_c}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_c \frac{\partial T_c}{\partial z} \right) = 0$$

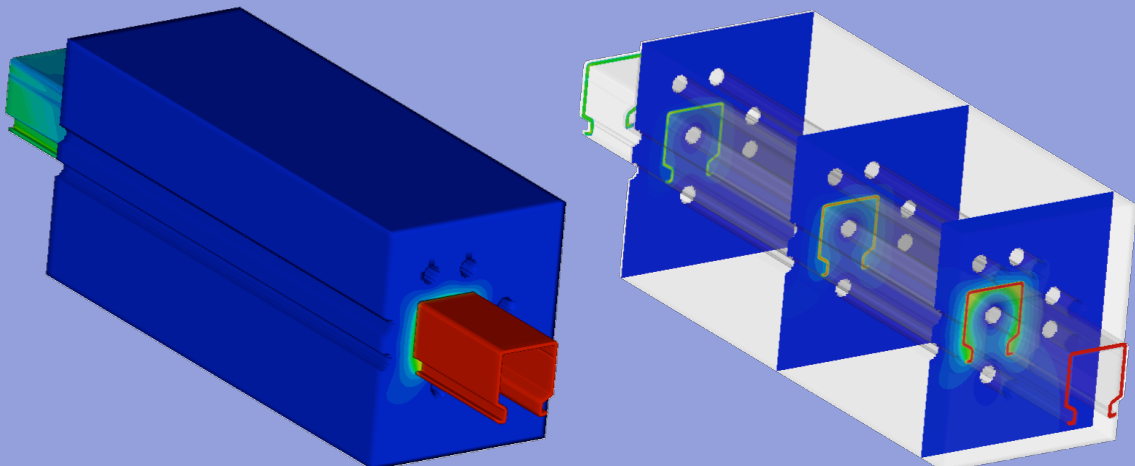
## Polymer-calibrator interface

*Contact Resistance*

$$k_c \left( \frac{\partial T_c}{\partial n} \right)_{\text{interface}} = -k_p \left( \frac{\partial T_p}{\partial n} \right)_{\text{interface}} = h_i (T_p - T_c)_{\text{interface}}$$



### 3D Temperature field calculation (FVM)





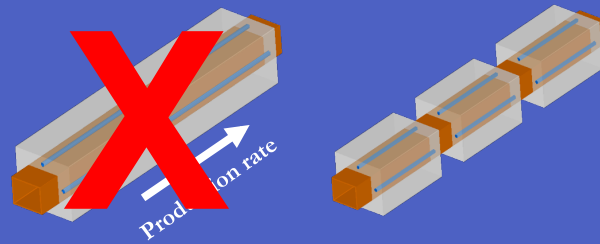
Influence of boundary conditions, process and geometrical parameters on the system performance (in terms of average temperature and temperature uniformity)

## Conclusion:

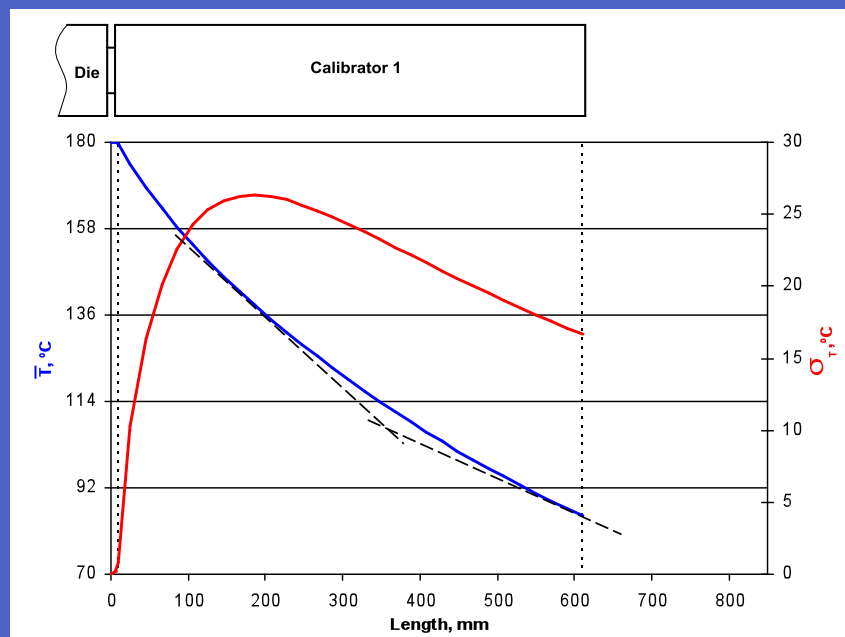
In general

$$\downarrow \bar{T} \Rightarrow \uparrow \sigma_T$$

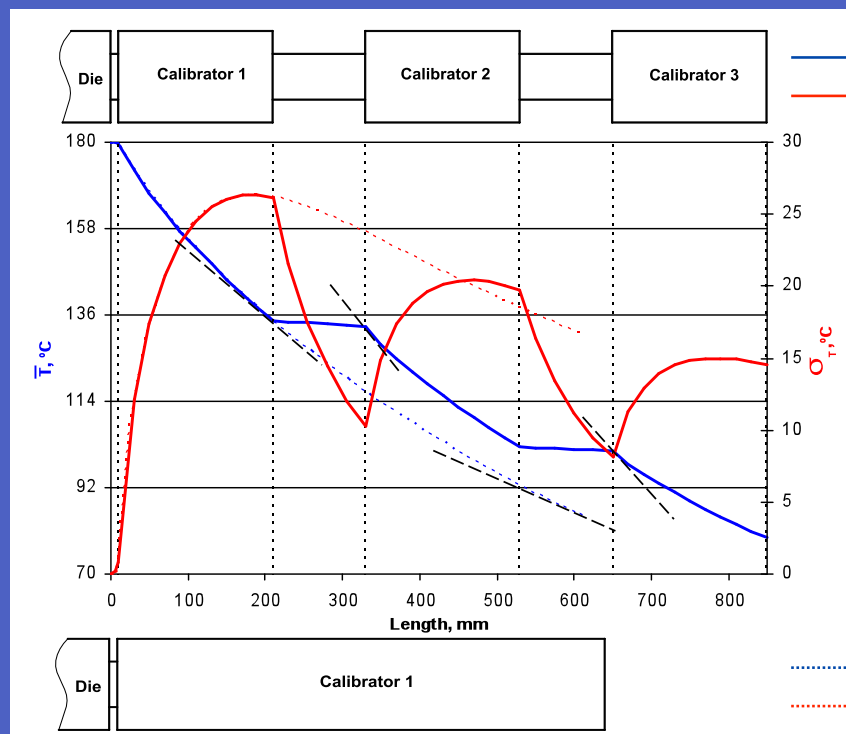
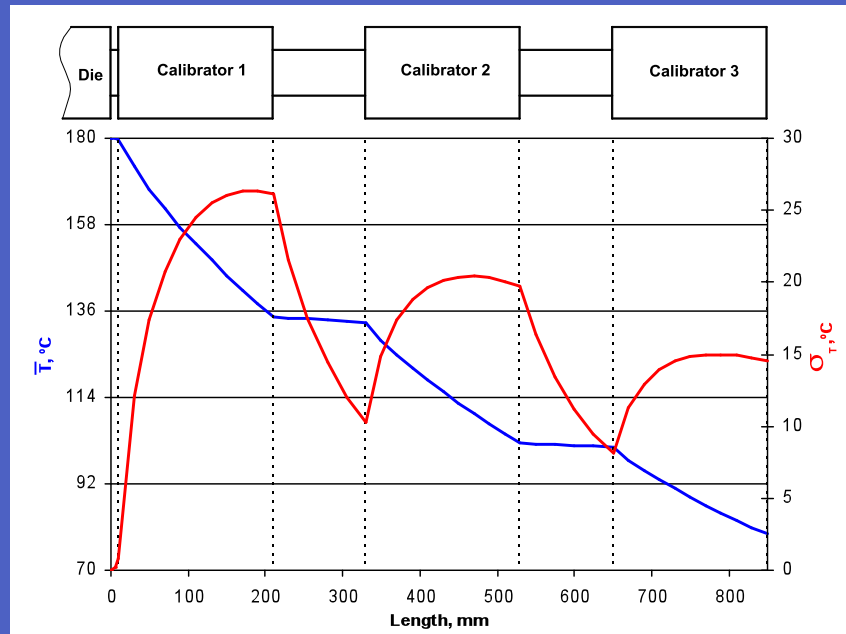
Exceptions



Polym Eng Sci, 44 (2004), p.2216-2228

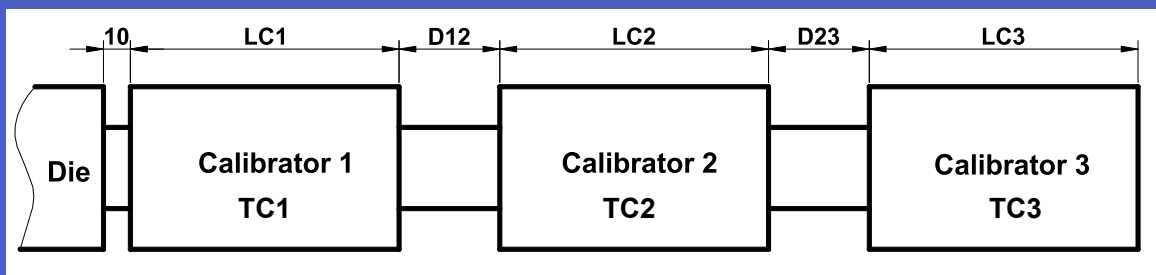




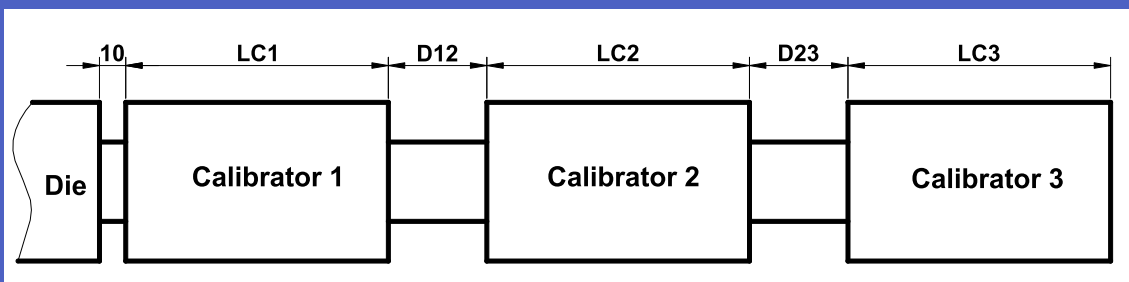




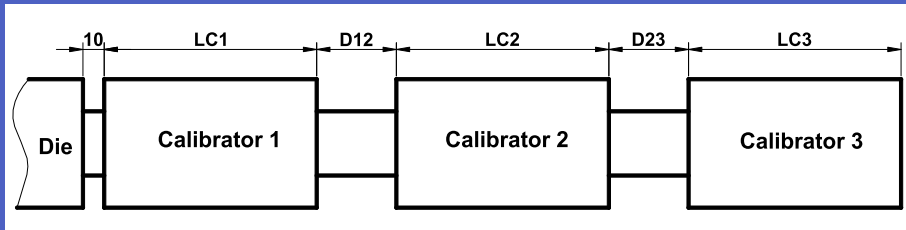
Influence of the cooling units and annealing zones lengths and cooling fluid temperature on the system performance



Influence of Length Distribution  
 $LC_i$  and  $D_{ij}$



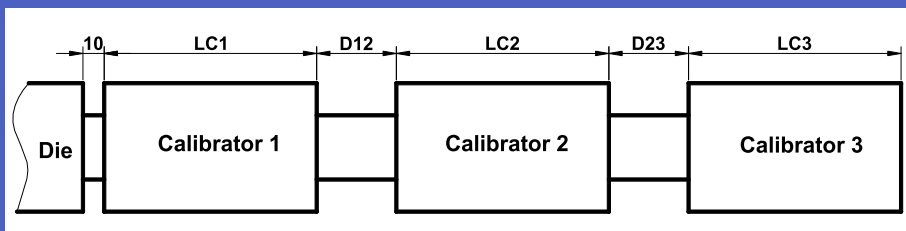
# Calibrators - System Behaviour



LCi	LC1	LC2	LC3	Dij	D12	D23
	[mm]	[mm]	[mm]		[mm]	[mm]
→	600	-	-	→	-	-
→	300	300	-	→	240	-
→	200	200	200	→	120	120
→	200	200	200	↗	60	180
→	200	200	200	↘	180	60
↘	300	200	100	→	120	120
↗	100	200	300	→	120	120
↘	300	200	100	↘	180	60
↘	300	200	100	↗	60	180
↗	100	200	300	↘	180	60
↗	100	200	300	↗	60	180

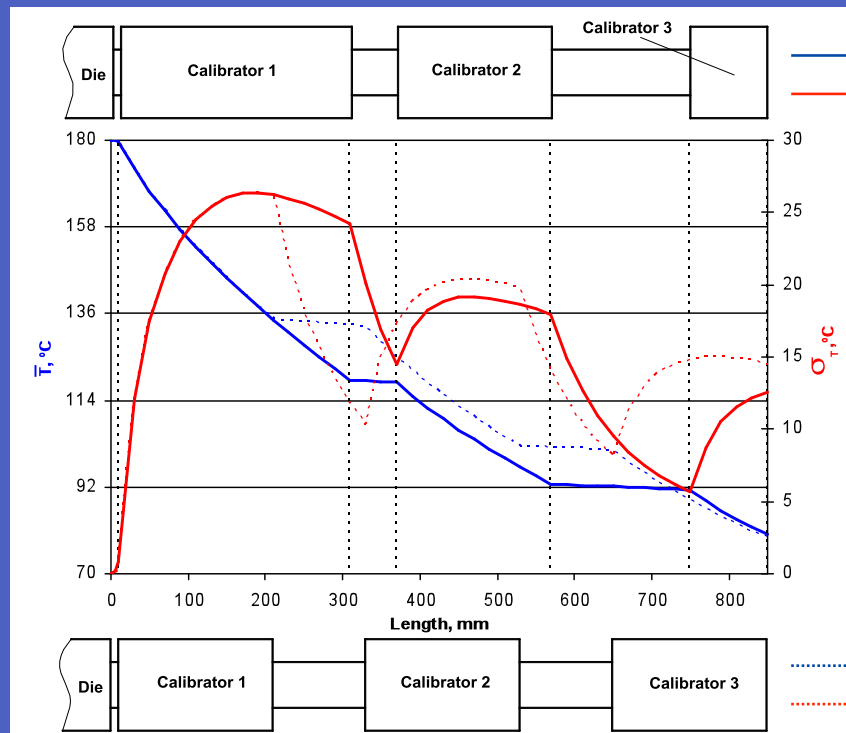
$\Sigma LC_i$  (600 mm),  $\Sigma D$  (240 mm) (system length = 850 mm)

# Calibrators - System Behaviour



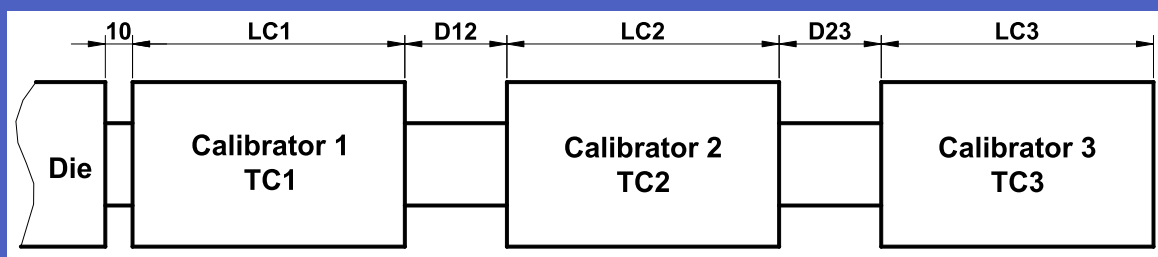
LCi	LC1	LC2	LC3	Dij	D12	D23	$\bar{T}$		$\sigma_T$	
	[mm]	[mm]	[mm]		[mm]	[mm]	[°C]	[%]	[°C]	[%]
→	600	-	-	→	-	-	84.9	0.0%	16.6	0.0%
→	300	300	-	→	240	-	80.3	-5.5%	15.2	-8.6%
→	200	200	200	→	120	120	79.2	-6.7%	14.5	-12.6%
→	200	200	200	↗	60	180	79.5	-6.4%	14.5	-13.1%
→	200	200	200	↘	180	60	79.4	-6.5%	14.8	-10.8%
↘	300	200	100	→	120	120	79.5	-6.4%	13.0	-22.1%
↗	100	200	300	→	120	120	79.4	-6.5%	15.1	-9.3%
↘	300	200	100	↘	180	60	79.6	-6.3%	13.8	-17.3%
↘	300	200	100	↗	60	180	79.9	-5.9%	12.6	-24.3%
↗	100	200	300	↘	180	60	79.7	-6.1%	15.2	-8.4%
↗	100	200	300	↗	60	180	79.5	-6.3%	15.1	-9.4%

$\Sigma LC_i$  (600 mm),  $\Sigma D$  (240 mm) (system length = 850 mm)



## Influence of cooling fluid temperature

$T_{Ci}$



# Calibrators - System Behaviour



TCi	TC1	TC2	TC3
	[°C]	[°C]	[°C]
	18	-	-
 → LCi + → Dij	18	18	18
	10	10	10
	26	26	26
	26	18	10
	10	18	26
 ↘ LCi + ↗ Dij	18	18	18
	10	10	10
	26	26	26
	26	18	10
	10	18	26

10°C ≤ TCi ≤ 26°C

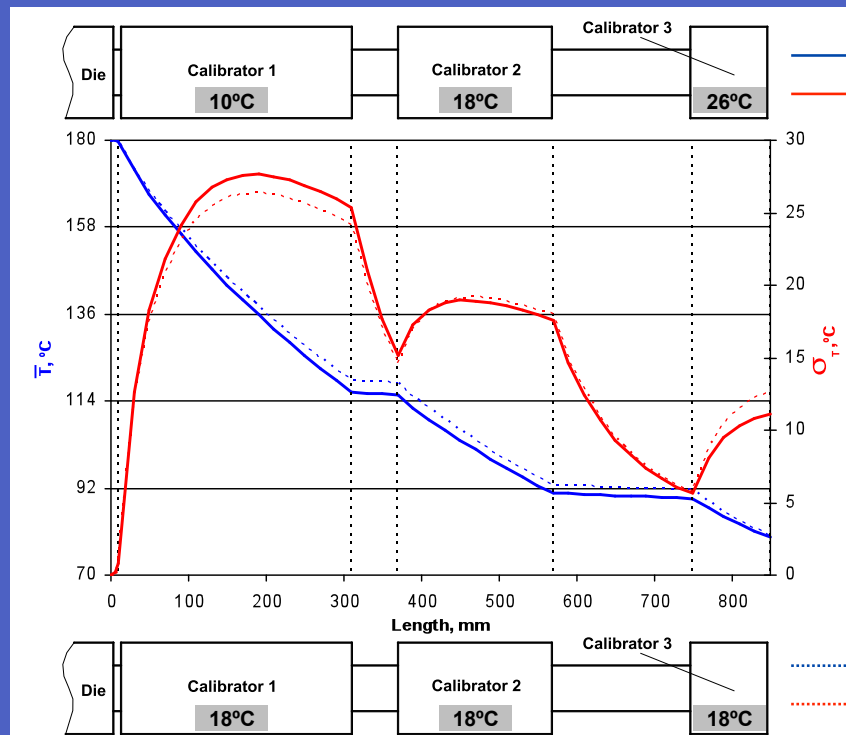
# Calibrators - System Behaviour



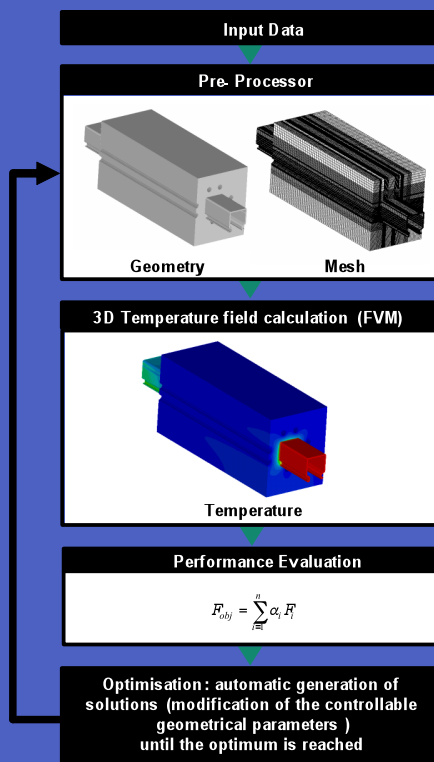
TCi	TC1	TC2	TC3	$\bar{T}$		$\sigma_T$	
	[°C]	[°C]	[°C]	[°C]	[%]	[°C]	[%]
	18	-	-	84.9	-	16.6	-
 → LCi + → Dij	18	18	18	79.2	-6.7%	14.5	-12.6%
	10	10	10	74.4	-12.3%	15.3	-8.2%
	26	26	26	83.9	-1.1%	13.8	-17.0%
	26	18	10	78.0	-8.1%	16.0	-3.6%
	10	18	26	80.3	-5.4%	13.0	-21.7%
 ↘ LCi + ↗ Dij	18	18	18	79.9	-5.9%	12.6	-24.3%
	10	10	10	75.2	-11.4%	13.2	-20.7%
	26	26	26	84.6	-0.4%	12.0	-28.0%
	26	18	10	80.2	-5.5%	14.1	-15.5%
	10	18	26	79.5	-6.3%	11.1	-33.1%

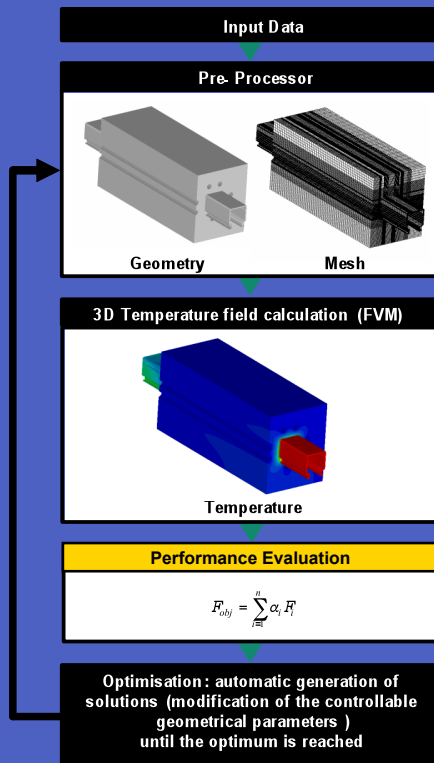
10°C ≤ TCi ≤ 26°C

# Calibrators - System Behaviour



# Calibrators - Optimisation Methodology





Temperature uniformity

$$\sigma_T = \sqrt{\frac{\sum_{i=1}^{n_i} (T_i - \bar{T})^2 A_i}{A_T}}$$

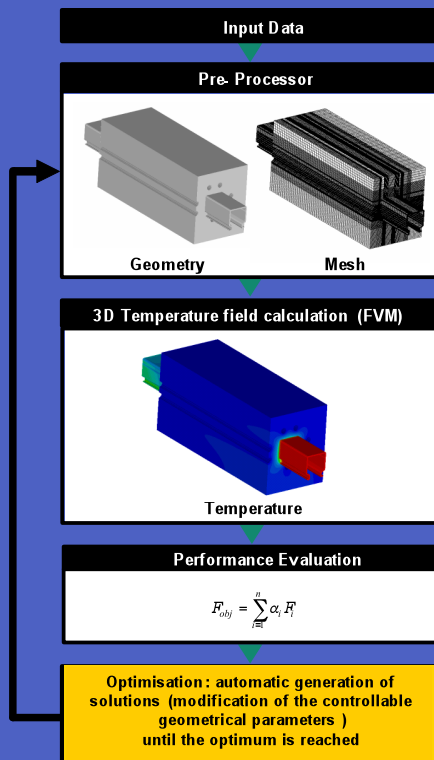
Cooling efficiency

$$\bar{T} = \frac{\sum_{i=1}^{n_i} T_i A_i}{A_T}$$

$$F_{obj} = K |\bar{T} - T_S| + \sigma_T$$

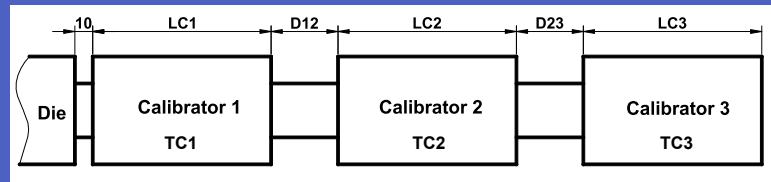
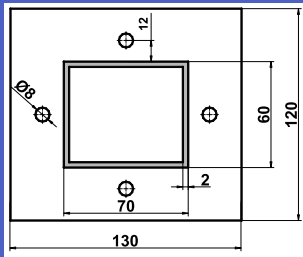
where:

$$\begin{cases} \bar{T} \leq T_S \Rightarrow K = 0 \\ \bar{T} > T_S \Rightarrow K = 1000 \end{cases}$$



## Optimisation algorithm

Non-linear SIMPLEX method



### Restrictions:

- Number of calibration/cooling units  $\leq 3$
- Total calibration length ( $\sum LC_i$ )  $\leq 600$  mm
- Total system length ( $\sum LC_i + \sum D_{ij} + 10$ )  $\leq 850$  mm
- Cooling Fluid Temperature  $TC_i \in [10^\circ\text{C}, 26^\circ\text{C}]$



### General conditions for the simulations

#### Processing conditions

$$v_p = 2 \text{ m/min}$$

$$T_m = 180 \text{ }^\circ\text{C}$$

$$T_f = 18 \text{ }^\circ\text{C}$$

$$T_s = 80 \text{ }^\circ\text{C}$$

#### Materials Properties

$$K_p = 0.18 \text{ W/mK}$$

$$K_c = 14 \text{ W/mK}$$

$$\rho_p = 1400 \text{ kg/m}^3$$

$$c_p = 1000 \text{ J/kgK}$$

#### Boundary conditions

Annealing zones: free convection and radiation

Polymer-calibrator interface: contact resistance

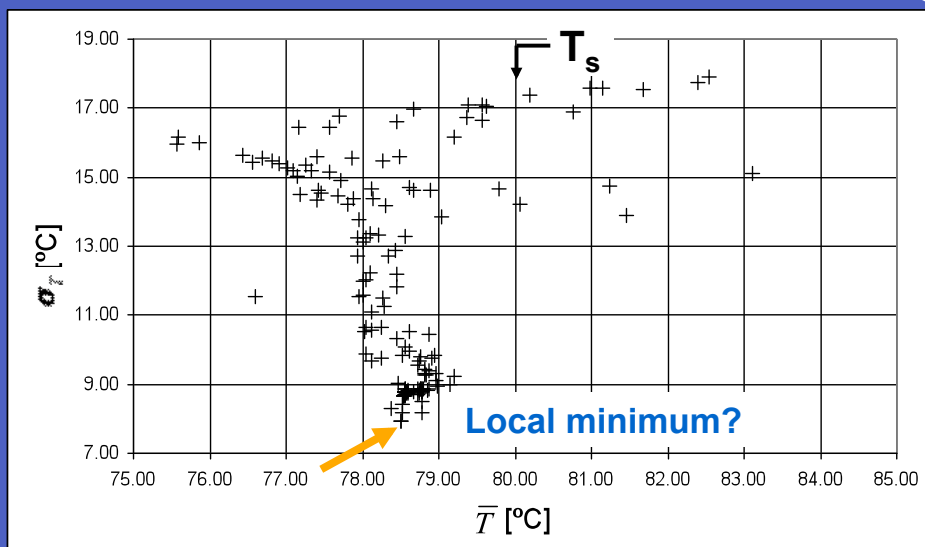
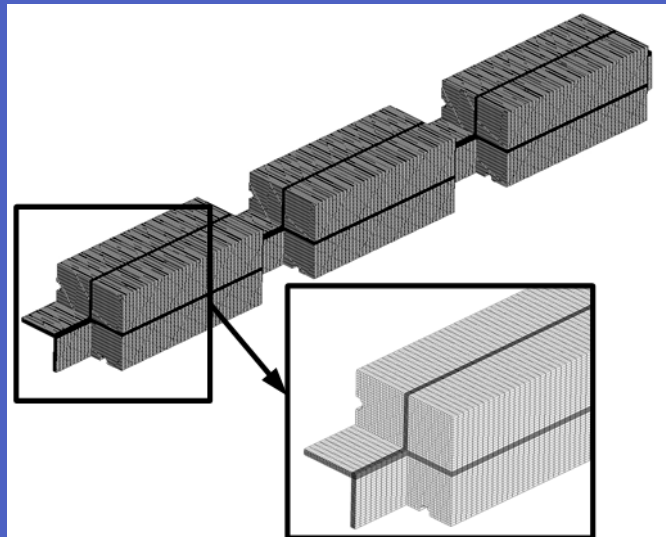
$$(h_i = 425 \text{ W/m}^2\text{K})$$



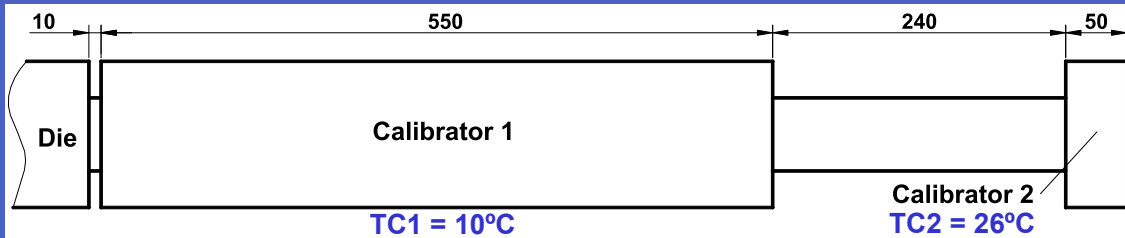
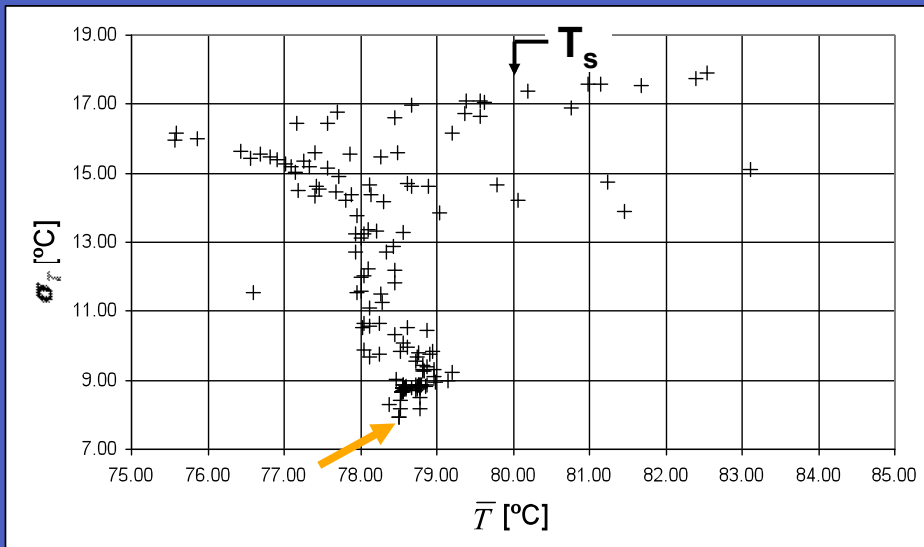


# Geometry

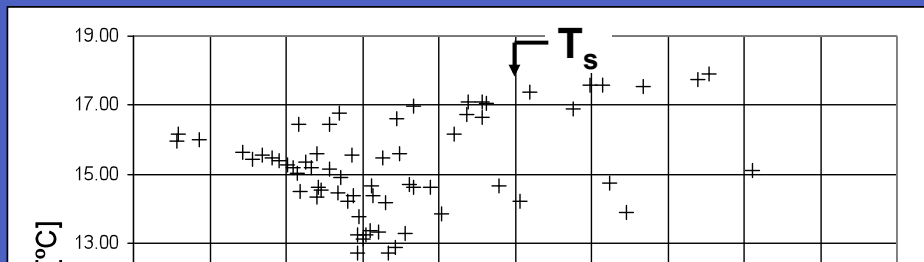
## Mesh



# Calibrators - Case Study

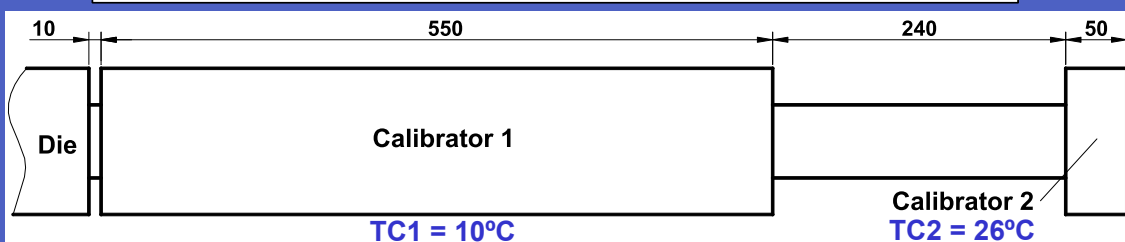


# Calibrators - Case Study



Geometry	$\bar{T}$ [°C]	$\sigma_T$ [°C]
One Calibrator (TC=18°C)	84.9	16.6
Optimum Solution	78.5	7.9

**- 52.4%**





- Cooling systems with
  - ascending cooling units lengths
  - descending annealing zone lengths
  - ascending cooling fluid temperaturesseem to have the best performance.



- The developed optimisation methodologies both for extrusion dies and calibrators were able to improve automatically the system performance;
- The optimisation methodologies are under development;
- The employment of numerical analysis allows a deeper insight of the process.

## Ongoing work

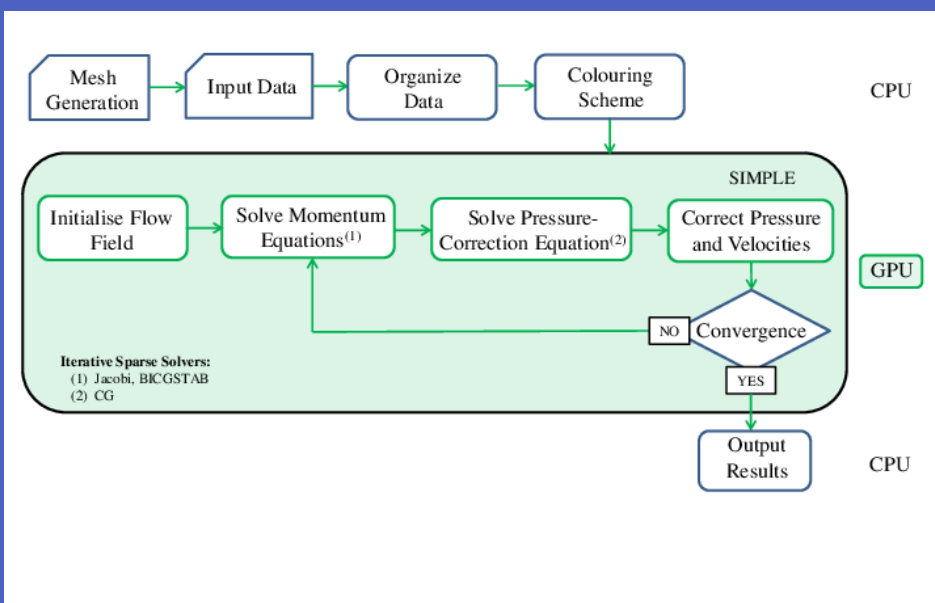


- Implementation of the **wall Slip** and **free-surface** boundary conditions (L.L. Ferrás, PhD project);
- Development of **unstructured** numerical modelling **code** (N.D. Gonçalves, PhD project);
- Prediction of **thermal** induced **stresses** (A.M. Ribau, FCT Research project);
- Development of **multiscale modelling** approaches (S.T. Mould, PhD Project);
- Development of **SPH** numerical modelling **code** (D.F. Cordeiro, PhD/Cooperation Project);

## Ongoing work



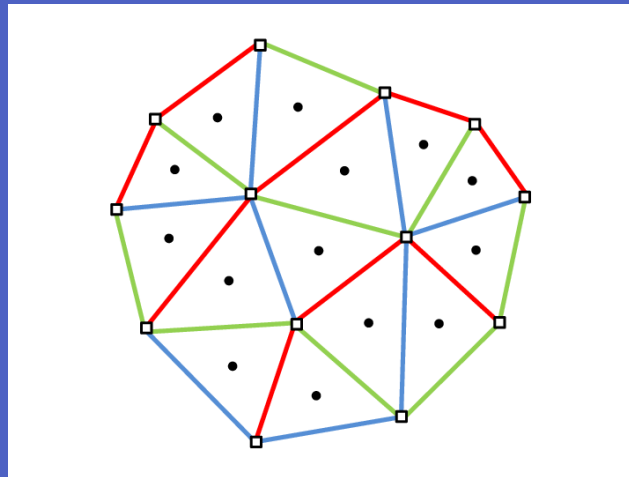
- Numerical code **parallelization** on **GPU** (S.P. Pereira, FCT Research Project);





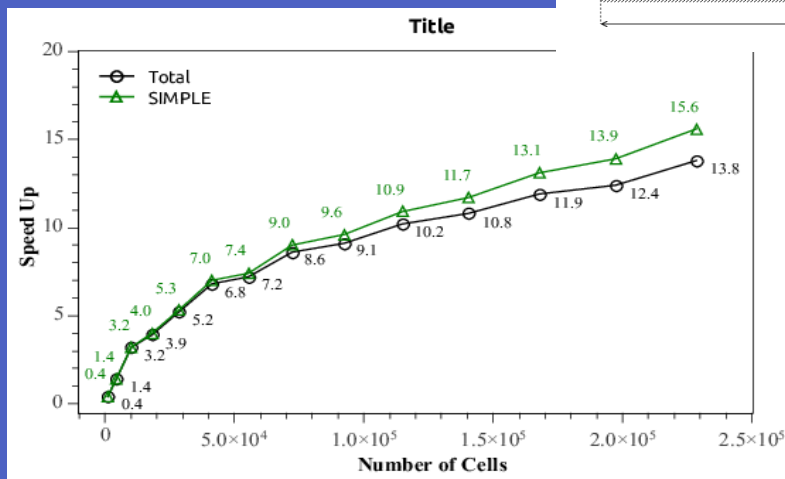
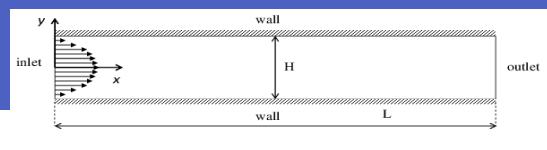
- Numerical code **paralelization** on **GPU** (S.P. Pereira, FCT Research Project);

A colouring scheme was used to avoid race conditions



- Numerical code **paralelization** on **GPU** (S.P. Pereira, FCT Research Project);

Poiseuille Flow – Speed Up





- Numerical code **parallelization** on **GPU** (S.P. Pereira, FCT Research Project);

## Lid Driven Cavity Flow – Speed Up

